F 3621 (Pages : 4) Reg. No.................................

Name.................................

B.TECH. DEGREE EXAMINATION, NOVEMBER 2014

Fifth Semester

Branch : Common to all Branches except C.S. and I.T.

EN 010 501-A—ENGINEERING MATHEMATICS—IV

(Regular/Improvement/Supplementary)

[New Scheme—2010 Admission onwards]

Time : Three Hours Maximum : 100 Marks

Part A

Answer **all** *questions.*

Each question carries 3 marks.

1. An electrostatic field in the xy-plane is given by the potential function $\phi = 3x^2y - y^3$, find the stream function.

2. Find the image of the circle $|z-1| = 1$ in the complex plane under the mapping $w = \frac{1}{z}$.

3. Find the real root of the equation $x^2 - 2x - 5 = 0$ by the method of false position correct to 3 decimal places.

4. Solve $\frac{dy}{dx} = 1 - y, y(0) = 0$ in the range $0 \le x \le 3$ by taking $h = 0.1$ by the modified Euler's method.

5. Construct the dual of the L.P.P.

   Maximize $z = 4x_1 + 9x_2 + 2x_3$

   subject to $2x_1 + 3x_2 + 2x_3 \le 7, 3x_1 - 2x_2 + 4x_3 = 5 ; x_1, x_2, x_3 \ge 0.$

(5 × 3 = 15 marks)

Part B

Answer **all** *questions.*

Each question carries 5 marks.

6. Show that $\sqrt{|xy|}$ is not analytic at the origin, although Cauchy-Riemann equations are satisfied at the point.

7. Find the Taylor's series expansion of $f(z) = \frac{2z^3 + 1}{z^2 + z}$ about $z = i$.

Turn over

8. Find by the iteration method, a real root of $2x - \log_{10}x = 7$.

9. Solve $\frac{dy}{dx} = x + z, \frac{dz}{dx} = x - y^2$ with $y(0) = 2$, $z(0) = 1$ to get $y(0.1)$, $y(0.2)$, $z(0.1)$ and $z(0.2)$ approximately by Taylor's series.

10. Using graphical method, solve the following L.P.P.

Maximize $z = 2x_1 + 3x_2$

subject to $x_1 - x_2 \le 2$

$x_1 + x_2 \ge 4,$

$x_1, x_2 \ge 0.$

(5 × 5 = 25 marks)

Part C

Answer **all** *questions.*
Each full question carries 12 *marks.*

11. (a) Determine the analytic function $f(z) = u + iv$ if $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f(\pi/2) = 0$.

(6 marks)

(b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| < 1$.

(6 marks)

Or

12. (a) Prove that the function $f(z)$ defined by $f(z) = \frac{x^3(1+i) - y^3 1 - i}{x^2 + y^2}$, $z \ne 0$ and $f(0) = 0$ is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.

(6 marks)

(b) Show that the transformation $w = \frac{3-z}{z-2}$ transforms the circle with center $\left(\frac{5}{2}, 0\right)$ and radius $\frac{1}{2}$ in the z-plane into the imaginary axis in the w-plane and the interior of the circle into the right half of the plane.

(6 marks)

13. (a) Evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$, where C is the circle (i) $|z| = 1$; (ii) $|z+1-i| = 2$; (iii) $|z+1+i| = 2$.

(8 marks)

(b) Determine the poles of the function $f(z) = \frac{x^2}{(z-1)^2 (z+2)}$ and the residue at each pole.

(4 marks)

Or

14. (a) Find the Laurent's expansion of $f(z)=\frac{7z-2}{(z+1)\,z\,(z-2)}$ in the region $1<|z+1|<3$. (5 marks)

(b) Show the method of residues, that $\int_0^{\pi} \frac{a}{a^2+\sin^2\theta}\,d\theta=\frac{\pi}{\sqrt{1+a^2}}$. (7 marks)

15. (a) Using Newton's iterative method, find the real root of $x \log_{10} x = 1.2$ correct to five decimal places.

(6 marks)

(b) Solve by Gauss-Seidel method :

$10x + 2y + z = 9$

$2x + 20y - 2z = -44$

$-2x + 3y + 10z = 22.$

(6 marks)

Or

16. (a) Find a real root of the equation $x^3 - x - 11 = 0$, correct to 4 decimal places using the bisection method.

(6 marks)

(b) Find the root of the equation $\cos x - xe^x = 0$ by secant method correct to four decimal places.

(6 marks)

17. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = yz + x$, $\frac{dz}{dx} = xz + y$ given that $y(0) = 1$, $z(0) = -1$ for $y(0.2)$, $z(0.2)$.

Or

18. Apply Milne's method, to find a solution of the differential equation $y' = x - y^2$ in the range $0 \le x \le 1$ for the boundary condition $y = 0$ at $x = 0$.

19. (a) What is the maximization transport problem ? How do you solve it ? (3 marks)

(b) Using simplex method solve the LPP

Maximize $z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \le 2$

$5x_1 + 2x_2 \le 10$

$3x_1 + 8x_2 \le 12,$

$x_1, x_2 \ge 0.$

(9 marks)

Or

Turn over

20. Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method (VAM). Here, F_1, F_2 and F_3 are factories, and W_1, W_2 and W_3 are warehouses.

	W_1	W_2	W_3	W_4	*Production of Factories*
F_1	21	16	25	13	11
F_2	17	18	14	23	13
F_3	32	27	18	41	19
Capacity of the warehouse	6	10	12	15	43

(5 × 12 = 60 marks)