F 3161 (Pages : 2) Reg. No.....

Name.....

B.TECH. DEGREE EXAMINATION, JANUARY 2007

Seventh Semester

Branch—Computer Science and Engineering

THEORY OF COMPUTATION (R)

(New Scheme—2002 admission onwards)

[Regular/Supplementary]

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

Part A

1. Prove that for any set A having $n \geq 0$ elements, the power set of A has 2^n elements.
2. Explain the types of functions - Injection, Surjection, Bijection and Invertible function.
3. Define regular expression.
4. Construct a DFA (transition diagram) accepting the language L = $\{w \in \{a,b\}^* \mid w$ has abab as substring$\}$.
5. Find a CFG for the language L = $\{a^n b^{2n} c^m \mid n,m \geq 0\}$.
6. Find a CFG for the language of all strings over $\{a, b\}$ with exactly one a or exactly one b.
7. What is a universal Turing machine?
8. Construct a Turing machine which computes the function $f(n) = n + 2$ over unary numbers.
9. What will happen when somebody finds a deterministic polynomial time algorithm for an NP - complete problem ? Comment on its consequence on the complexity classes.
10. Define class P and class NP.

(10 × 4 = 40 marks)

Part B

11. (a) Prove that the set of real numbers is uncountable. (6 marks)

    (b) Given that A and B are subsets of a universal set U. Prove that

    (i) $A - B = A \cap B'$

    (ii) $A - B = A$ if and only if $A \cap B = \varnothing$

    (iii) $A - B = \varnothing$ if and only if $A \subseteq B$.

    (6 marks)

Or

12. (a) Explain the Chomsky hierarchy of formal languages. (8 marks)

    (b) Define 'formal language'. Give examples for finite and infinite languages. (4 marks)

Turn over

13. (a) Prove that L={$a^n b^n$ | $n > 0$} is not regular. (6 marks)

(b) Prove that L={a^s 1^s is a perfect square} is not regular. (6 marks)

Or

14. (a) State and prove Pumping lemma for regular languages. (8 Marks)

(b) Construct a DFA accepting L = { $w \in \{a,b\}^*$ | each 'a' (if one is there) in w is immediately preceded and immediately followed by a 'b' }. (4 marks)

15. (a) Remove all unit productions, useless productions and ε - productions from the grammar :

$S \rightarrow aA \mid aBB$

$A \rightarrow aaA \mid \varepsilon$

$B \rightarrow bB \mid bbC$

$C \rightarrow B$

What language does this grammar generate ? (7 marks)

(b) Convert the following grammar into Chomsky normal form :

$S \rightarrow ABa$

$A \rightarrow aaB$

$B \rightarrow Ac.$

(5 marks)

Or

16. Prove that for any context free language L there exist a non-deterministic PDA, M such that L = L (M). (12 marks)

17. (a) Construct a Turing machine that decides the language L = { $a^n b^n \mid n \geq 0$ }. (8 marks)

(b) Construct a Turing machine that shifts the input string one position to the left. (4 marks)

Or

18. (a) Differentiate between recursive and recursively enumerable languages. (4 marks)

(b) Construct a Turing machine that accepts the language L = { $w \in \{a,b\}^*$ | w has equal no. of a's and b's }. (8 marks)

19. Prove that 3SAT problem is NP-complete.

Or

20. State Cook's theorem and give an outline of its proof. (12 marks)

[5 × 12 = 60 marks]