APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.Tech DEGREE REGULAR EXAMINATION DECEMBER (2024)

SCHEME OF 0360YMAT101122402

GYMAT101 – MATHEMATICS FOR ELECTRICAL SCIENCE – I.

PART A.

1. $$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix} \quad R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 3 & -2 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\sim \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank $= 2$

(Elementary Linear Algebra Howard Anton, Chris Rorres Wiley 11th edition, 2019)

2. $$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0 \implies \begin{vmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{vmatrix} = 0.$$

$$(1-\lambda)(3-\lambda) = 0$$

$$\lambda^2 - 4\lambda + 3 = 0.$$

$$\lambda = 1, 3.$$

When $\lambda = 1$, $[A - \lambda I]X = 0$

$$\begin{bmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$0x_1 + 2x_2 = 0.$

$x_1 = 1, \quad x_2 = 0.$

Eigen vector corresponding to $\lambda = 1$ is

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

When $\lambda = 3$, $[A - \lambda I]X = 0.$

$$\begin{bmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

$-2x_1 + 2x_2 = 0.$

$x_1 = 1, \quad x_2 = 1$

Eigen vector corresponding to $\lambda = 3$ is

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(Elementary Linear Algebra Howard Anton, Chris Rorres wiley, 11th edn, 2019)

3. $y_1 = e^{-2x}, \quad y_2 = e^{3x}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^{-2x} & e^{3x} \\ -2e^{-2x} & 3e^{3x} \end{vmatrix}$$

$$= e^{-2x} \times 3 \times e^{3x} - -2e^{-2x}e^{3x}$$

$$= 5e^{-2x}e^{3x}$$

$$= 5e^{x}$$

$$\neq 0.$$

(Advanced Engineering Mathematics Erwin Kreyszig)

$\therefore$ y_1 and y_2 are linearly independent.

4) $y'' + 3y = 0$

$\lambda^2 + 3 = 0$

$\lambda = \pm\sqrt{3}\, i$

$\therefore y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

$= c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x$

(Advanced Engineering Mathematics Erwin Kreyszig)

5) $L[e^{2t} + 4t^3 - 2\sin 3t)$

$= L[e^{2t}] + 4L(t^3) - 2L(\sin 3t)$

$= \frac{1}{s-2} + 4 \cdot \frac{3!}{s^4} - 2 \cdot \frac{3}{s^2+3^2}$

$= \frac{1}{s-2} + \frac{24}{s^4} - \frac{6}{s^2+9}$

(Bird's Higher Engineering Mathematics John Bird Taylor & Francis, 9th ed)

6) $L^{-1}\left[\frac{s^2-3s+4}{s^3}\right] = L^{-1}\left[\frac{1}{s} - \frac{3}{s^2} + \frac{4}{s^3}\right]$

$= L^{-1}\left[\frac{1}{s}\right] - 3L^{-1}\left[\frac{1}{s^2}\right] + 4L^{-1}\left[\frac{1}{s^3}\right]$

$= 1 - 3t + \frac{4t^2}{2!}$

$= 1 - 3t + 2t^2$

7) $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx.$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx.$ (Bird's Higher Engineering Mathematics, John Bird, Taylor & Francis, 9th e)

8) $f(x) = e^x$ $\quad f(0) = e^0 = 1$

$f'(x) = e^x$ $\quad f'(0) = e^0 = 1$

$f''(x) = e^x$ $\quad f''(0) = e^0 = 1$

$f'''(x) = e^x$ $\quad f'''(0) = e^0 = 1$

$\vdots$

$\therefore$ Maclaurin's series for $f(x) =$

$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$

$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

PART B.

MODULE I. (Elementary Linear Algebra, Howard Anton, Chris Rorres, Wiley, 11e)

9) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 5 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \mu \\ \mu^2 \end{bmatrix}$

$x+y+z=1$

$x-2t+t=1$

$x-t=1$

$x=1+t$

$\mu=1 \Rightarrow x=1+t, \quad y=-2t, \quad z=t.$

When $\mu=3$:

choose $z=t$

$y+2z=\mu-1$

$\Rightarrow y+2t=3-1$

$\Rightarrow y+2t=2$

$\Rightarrow y=2-2t$

$x+y+z=1$

$x+2-2t+t=1$

$x+2-t=1$

$x=1-2+t$

$x=t-1$

$\mu=3 \Rightarrow \underline{\underline{x=t-1, \quad y=2-2t}}, \quad z=t$

10) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

Characteristic equation is

$\lambda^3-6\lambda^2+11\lambda-6=0.$

$\therefore \lambda=1,2,3.$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & \mu \\ 1 & 5 & 9 & \mu^2 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & \mu-1 \\ 0 & 4 & 8 & \mu^2-1 \end{bmatrix} \quad R_3 \to R_3 - 4R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & \mu-1 \\ 0 & 0 & 0 & \mu^2-4\mu+3 \end{bmatrix}$$

$\therefore$ The system is consistent

if Rank $[AB]$ = Rank $[A]$.

This is possible only if

$\mu^2 - 4\mu + 3 = 0$.

$\mu = 1, 3$.

$x + y + z = 1$

$y + 2z = \mu - 1$

$n = 3$, $r = 2$, $n - r = 3 - 2 = 1$.

when $\mu = 1$

choose $z = t$

$y + 2z = \mu - 1$

$\Rightarrow y + 2t = 1 - 1$

$\Rightarrow y + 2t = 0$.

$\Rightarrow y = -2t$.

When $\lambda = 1$, $[A - \lambda I]X = 0$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad R_3 \rightarrow R_3 - R_2$$

$$\sim \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$0x_1 + 2x_2 + 2x_3 = 0$.

$0x_1 + 0x_2 + x_3 = 0$.

Eigen vector corresponding to $\lambda = 1$ is

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

When $\lambda = 2$, $[A - \lambda I]X = 0$.

$$\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-x_1 + 0x_2 + x_3 = 0$.

$0x_1 + x_2 + 2x_3 = 0$.

Eigen vector corresponding to $\lambda = 2$ is

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

when $\lambda = 3$, $[A - \lambda I] X = 0$.

$$\begin{bmatrix} -2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad R_3 \rightarrow 2R_3 + R_2$$

$$\sim \begin{bmatrix} -2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-2x_1 + 0x_2 + x_3 = 0$.

$0x_1 + 0x_2 + 2x_3 = 0$.

Eigen vector corresponding to $\lambda = 3$ is

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$D = P^{-1} A P$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

MODULE 2. (Advanced Engineering Mathematics, Erwin Kreyszig. John wiley & sons 10 e)

11. $y'' - y = x^2$.

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$.

$\therefore y_h = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

$= c_1 e^{x} + c_2 e^{-x}$

$y_1 = e^x$, $y_2 = e^{-x}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$$

$$= e^x \times -e^{-x} - e^x e^{-x}$$

$$= -e^0 - e^0$$

$$= -2.$$

$$y_p = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx.$$

$$= -e^x \int \frac{e^{-x} x^2}{-2} dx + e^{-x} \int \frac{e^x \cdot x^2}{-2} dx.$$

$$= \frac{-e^x}{-2} \int e^{-x} x^2 dx - \frac{e^{-x}}{2} \int e^x . x^2 dx.$$

$$= \frac{e^x}{2} \left[x^2 . \frac{e^{-x}}{-1} - 2x e^{-x} + \frac{2 e^{-x}}{-1} \right]$$

$$- \frac{e^{-x}}{2} \left[x^2 e^x - 2x e^x + 2 e^x \right]$$

$$= \frac{e^x}{2} \left[-x^2 e^{-x} - 2x e^{-x} - 2 e^{-x} \right] - \frac{e^{-x}}{2} \left[x^2 e^x - 2x e^x + 2 e^x \right]$$

$$= -\frac{x^2}{2} - x - 1 - \frac{x^2}{2} + x - 1$$

$$= \underline{-x^2 - 2}$$

or $y = y_h + y_p$

$= c_1 e^{x} + c_2 e^{-x} - x^2 - 2$

12 a) $y'' + 4y = 0$.

$\lambda^2 + 4 = 0$

$\lambda = \pm 2i$

$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$.

$= c_1 \cos 2x + c_2 \sin 2x$.

$y(0) = 4 \Rightarrow c_1 \cos 0 + c_2 \sin 0 = 4$

$\Rightarrow c_1 = 4$.

$y' = c_1 - \sin 2x \times 2 + c_2 \cos 2x \times 2$.

$y'(0) = 2 \Rightarrow -2c_1 \sin 0 + 2c_2 \cos 0 = 2$.

$\Rightarrow 2c_2 = 2$.

$\Rightarrow c_2 = 1$.

$\therefore y = 4\cos 2x + \sin 2x$

12 b) $y'' + 3y' + 2y = 12x^2$.

$\lambda^2 + 3\lambda + 2 = 0$.

$\lambda = -1, -2$.

$y_h = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

$= c_1 e^{-x} + c_2 e^{-2x}$.

$$y_p = Ax^2 + Bx + C$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A.$$

$$2A + 3(2Ax + B) + 2(Ax^2 + Bx + C) = 12x^2.$$

Equating coefficient of x^2

$$2A = 12$$

$$\Rightarrow A = \frac{12}{2} = \underline{\underline{6}}.$$

Equating coefficient of x.

$$6A + 2B = 0$$

$$\Rightarrow 6 \times 6 + 2B = 0$$

$$\Rightarrow 36 + 2B = 0$$

$$\Rightarrow 2B = -36$$

$$\Rightarrow B = \frac{-36}{2} = \underline{\underline{-18}}.$$

Equating constants.

$$2A + 2B + 2C = 0.$$

$$\Rightarrow 12 - 54 + 2C = 0$$

$$\Rightarrow 2C = 54 - 12 = 42$$

$$\Rightarrow C = \frac{42}{2} = \underline{\underline{21}}.$$

$$\therefore y_p = 6x^2 - 18x + 21.$$

$$\therefore y = y_h + y_p$$

$$= c_1 e^{-x} + c_2 e^{-2x} + 6x^2 - 18x + 21$$

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MODULE 3

(Bird's Higher engineering Mathematics, John Bird Taylor & Francis 9e)

9) $y'' + 5y' + 6y = 0.$

$L(y'') + 5L(y') + 6L(y) = L(0).$

$s^2L(y) - sy(0) - y'(0) + 5sL(y) - 5y(0) + 6L(y) = 0$

$L(y)(s^2 + 5s + 6) + 1 = 0.$

$$L(y) = \frac{-1}{s^2+5s+6}.$$

$$= \frac{-1}{(s+3)(s+2)}$$

$$\frac{-1}{(s+3)(s+2)} = \frac{A}{s+3} + \frac{B}{s+2}.$$

$$-1 = A(s+2) + B(s+3)$$

put $s = -2$.

$-1 = B.$

$\Rightarrow B = \underline{\underline{-1}}.$

put $s = -3$,

$-1 = A(-1)$

$\Rightarrow A = \underline{\underline{1}}.$

$$\therefore L(y) = \frac{1}{s+3} - \frac{1}{s+2}.$$

$$y = L^{-1}\left(\frac{1}{s+3} - \frac{1}{s+2}\right)$$

$$= e^{-3t} - e^{-2t}$$

13.b) $L^{-1}\left[\dfrac{2s+1}{s^2+2s+5}\right] = L^{-1}\left[\dfrac{2s+1+1-1}{s^2+2s+5+1-1}\right]$

$$= L^{-1}\left[\frac{2s+2-1}{s^2+2s+1+4}\right]$$

$$= L^{-1}\left[\frac{2(s+1)-1}{(s+1)^2+2^2}\right]$$

$$= L^{-1}\left[\frac{2(s+1)}{(s+1)^2+2^2}\right] - L^{-1}\left[\frac{1}{(s+1)^2+2^2}\right]$$

$$= 2e^{-t}\cos 2t - e^{-t}\frac{\sin 2t}{2}.$$

14 a) $L^{-1}\left[\dfrac{1}{s(s^2+4)}\right] = L^{-1}\left[\dfrac{1}{s}\right] * L^{-1}\left[\dfrac{1}{s^2+4}\right]$

$$= 1 \times \frac{\sin 2t}{2}.$$

$$= \int_0^t \frac{\sin 2u}{2}\,du.$$

$$= \frac{1}{2}\left[-\frac{\cos 2u}{2}\right]_0^t$$

$$= \frac{1}{2}\left[-\frac{\cos 2t}{2} + \frac{\cos 0}{2}\right]$$

$$= \frac{1}{2}\left[-\frac{\cos 2t}{2} + \frac{1}{2}\right] = \frac{1-\cos 2t}{4}.$$

14 (b)

$$L[f(t)] = \int_{1}^{2} e^{-st}(t-1)\,dt + \int_{2}^{3} e^{-st}(3-t)\,dt$$

$$= \left[(t-1)\frac{e^{-st}}{-s} - (1-0)\frac{e^{-st}}{s^2}\right]_1^2 + \left[(3-t)\frac{e^{-st}}{-s} - (-1)\frac{e^{-st}}{s^2}\right]_2^3$$

$$= (2-1)\frac{e^{-2s}}{\cancel{-s}} - \frac{e^{-2s}}{s^2} + \frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s^2}$$

$$+ \frac{e^{-2s}}{\cancel{s}} - \frac{e^{-2s}}{s^2}.$$

$$= -\frac{2e^{-2s}}{s^2} + \frac{e^{-s}}{s^2} + \frac{e^{-3s}}{s^2}$$

$$= \frac{e^{-s}}{s^2} - \frac{2e^{-2s}}{s^2} + \frac{e^{-3s}}{s^2}$$

MODULE 4. (Bird's Higher Engineering Mathematics, John Bird, Taylor & Francis 9e)

15.

$$S(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}.$$

$$a_0 = \frac{1}{2l}\int_{-l}^{l} f(x)\,dx.$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi x}{l}\,dx.$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} \, dx.$$

$$a_0 = \frac{1}{2l} \int_{-l}^{l} f(x) \, dx$$

$$= \frac{1}{4} \int_{-2}^{2} (x^2 - 2) \, dx$$

$$= \frac{1}{4} \left[\frac{x^3}{3} - 2x \right]_{-2}^{2}$$

$$= \frac{1}{4} \left[\frac{8}{3} - 4 + \frac{8}{3} - 4 \right]$$

$$= \frac{1}{4} \left[\frac{16}{3} - 8 \right] = \frac{1}{4} \left[\frac{16 - 24}{3} \right]$$

$$= \frac{1}{4} \left[\frac{-8}{3} \right]$$

$$= \frac{-2}{3}.$$

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} \, dx.$$

$$= \frac{1}{2} \int_{-2}^{2} (x^2 - 2) \cos \frac{n\pi x}{2} \, dx.$$

$$= \frac{1}{2} \left[(x^2 - 2) \frac{\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)} - (2x) \frac{-\cos\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^2} + 2 \frac{\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)^3} \right]_{-2}^{2}$$

$$= \frac{1}{2}\left[\frac{2x \cos\frac{n\pi x}{2}}{\frac{n^2\pi^2}{4}}\right]_{-2}^{2}$$

$$= \frac{1}{2}\left[\frac{4\cos n\pi + 4\cos n\pi}{\frac{n^2\pi^2}{4}}\right]$$

$$= \frac{4}{2}\left[\frac{8\cos n\pi}{n^2\pi^2}\right]$$

$$= \frac{16\cos n\pi}{n^2\pi^2}$$

$$= \frac{16(-1)^n}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi x}{l}\,dx.$$

$$= \frac{1}{2}\int_{-2}^{2}(x^2-2)\sin\frac{n\pi x}{2}\,dx.$$

$$= \frac{1}{2}\left[(x^2-2)\frac{-\cos\frac{n\pi x}{2}}{n\pi} - (2x-0)\frac{-\sin\frac{n\pi x}{2}}{n^2\pi^2} + 2\frac{\cos\frac{n\pi x}{2}}{n^3\pi^3}\right]_{-2}^{2}$$

$$= \frac{1}{2}\left[-(x^2-2)\frac{\cos\frac{n\pi x}{2}}{n\pi} + 2\frac{\cos\frac{n\pi x}{2}}{n^3\pi^3}\right]_{-2}^{2}$$

$$= \frac{1}{2}\left[-2\frac{\cos n\pi}{n\pi} + 2\frac{\cos n\pi}{n^3\pi^3} + 2\frac{\cos n\pi}{n\pi} - 2\frac{\cos n\pi}{n^3\pi^3}\right]$$

$$= \frac{1}{2}\times 0$$

$$= \underline{\underline{0}}$$

$$\therefore S(x) = -\frac{2}{3} + \sum_{n=1}^{\infty} \frac{16}{n^2\pi^2}(-1)^n \cos\frac{n\pi x}{2}$$

16. $a_0 = \frac{1}{L}\int_0^L f(x)\,dx.$

$$= \frac{1}{\pi}\int_0^{\pi} x\,dx. = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2}\right] = \underline{\underline{\frac{\pi}{2}}}.$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx.$$

$$= \frac{2}{\pi}\int_0^{\pi} x\cos\frac{n\pi x}{\pi}\,dx.$$

$$= \frac{2}{\pi}\left[x.\frac{\sin nx}{n} - 1.\frac{-\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos 0}{n^2}\right].$$

$$= \frac{2}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right] = \frac{2}{n^2\pi}\left[(-1)^n - 1\right].$$

$$= \begin{cases} \frac{-4}{n^2\pi}, & n \text{ is odd.} \\ 0, & n \text{ is even.} \end{cases}$$

$$S(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L}.$$

$$= \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left((-1)^n - 1\right)\cos nx.$$

$$= \frac{\pi}{2} - \frac{4}{\pi}\left[\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \right.$$

Since $f(x) = x$ is continuous at 0,

$$S(0) = f(0).$$

$$\frac{\pi}{2} - \frac{4}{\pi}\left[\cos 0 + \frac{1}{3^2}\cos 0 + \frac{1}{5^2}\cos 0 + \cdots\right] = 0$$

$$\frac{\pi}{2} - \frac{4}{\pi}\left[1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right] = 0.$$

$$-\frac{4}{\pi}\left[1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right] = \frac{-\pi}{2}.$$

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$