G 6554

M.TECH. DEGREE EXAMINATION, MAR 2011
Second Semester

Branch : Electrical and Electronics Engg.
Specialization : Power Electronics and Power Systems

PEPS 206.2 POWER SYSTEM PLANNING AND RELIABILITY (ELECTIVE-III)

Time: Three Hours **Maximum: 100 Marks**

Answer any five questions.
All questions carry equal marks.

1. (a) Discuss the objectives of planning in power systems. Describe long and short term planning. [10 Marks]

   (b) What are the basic characteristics of loads? [10 Marks]

2. (a) Describe the methodology of forecasting in detail. [8 Marks]

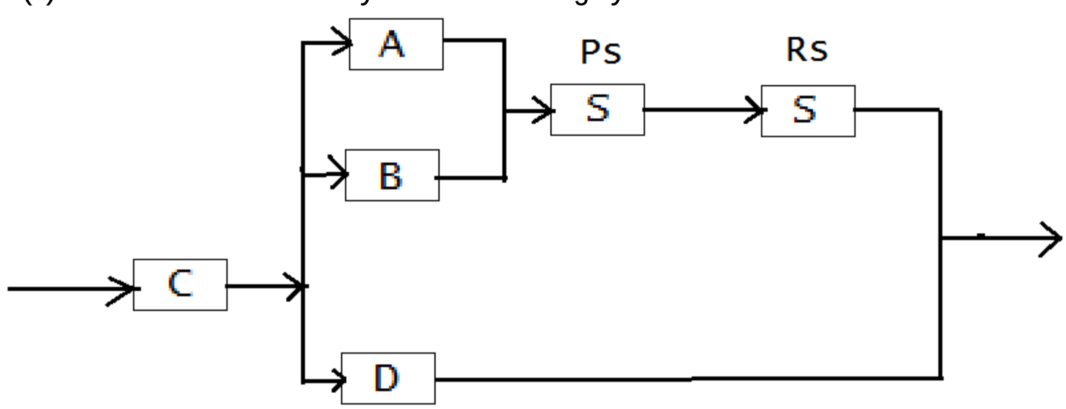
   (b) Forecast the peak load in 2015 using a linear characteristic. [12 Marks]

Year	2009	2010	2011	2012	2013	2014
Peak Demand (MW)	71	72	79	81	90	93

3. (a) How will you determine the reliability in series and parallel systems? [10 Marks]

   (b) Explain the probability models for generating units and loads. [10 Marks]

4. (a) Evaluate the reliability of the following system. [10 Marks]



Ra=0.9 Rb=0.96 Rc=0.99 Rd=0.8 Ps=0.92 Rs=0.98

   (b) Assume a generating system consisting of the following machines with their associated outage rates.

MW	Outage rate
10	0.02
10	0.02
10	0.02
10	0.02
5	0.02

Compute the probability outage table for the first four units. [10 Marks]

5. (a) Describe how transmission system reliability can be analyzed. [10 Marks]

(b) Describe with a typical example the frequency and duration method. [10 Marks]

6. (a) Give a short note on two plant style load system. [8 Marks]

(b) Two power systems are interconnected by a 20 MW tie-line. System A has three 20 MW generating units with forced outage rates of 10%. System B has two 30 MW units with forced outage rates of 20%. Calculate the LOLE in system A for one-day period, given that the peak load in both system A and system B is 30 MW. [12 Marks]

7. Write short notes on the following.

(a) Loss of load approach [7 Marks]
(b) Frequency and duration approach [7 Marks]
(c) Multiple bridge equivalents [6 Marks]

[5 x 20 = 100 Marks]

1. (a) Discuss the objectives of planning in power systems. Describe long and short term planning. [10 Marks]

Ref : 1) "Power System Reliability, Safety and Management", Balbir Singh Dhillon, Ann Arbor Science, -Page 147
2) "Power System Planning", R.L.Sullivan, McGraw-Hill, -Page 18

Power system planning is a fundamental topic; not to be confused with power system expansion planning. Power system expansion planning is a subset of power system planning.

The main objectives of power system planning are

i) To maintain future power generation and transportation costs.
ii) To increase the electric power system reliability according to specified conditions; system planning should be accomplished so that power system reliability is increased within the specified voltage and frequency required by the consumer.
iii) To predict the future electric energy requirements. This is often referred to as load forecasting.
iv) To plan for future electric power systems.

1. (b) What are the basic characteristics of loads? **[10 Marks]**

Ref : 1) "Power System Planning", R.L.Sullivan, McGraw-Hill, -Page 18

Electrical loads are broadly classified into

1) Residential
Residential customers use energy for domestic purposes.
Residential customers can be subdivided into rural and urban categories.

2) Commercial
Commercial customers use energy for business and trade purposes.

3) Industrial
Industrial customers use energy for manufacturing and value-added services; beginning from primary and Greenfield projects right up to the finishing stage.

4) Others (Municipalities, Boards, Public authorities)
This category of customers such as city corporations uses energy for street/highway lighting, public utility purposes, such as pumping water.

Characteristics of loads

Residential category

Has the most constant annual growth rate.
Widespread usage of weather sensitive loads such as space heaters, water heaters, air conditioners and refrigerators.
Has the most seasonal fluctuations. This is responsible for the seasonal variations in the system peak load.
System load patterns will be dependent upon the per capita consumption due to increase in weather sensitive residential loads.
Affected by demographic patterns.
Affected by economic patterns.

Commercial customers

Widespread usage of weather sensitive loads such as space heaters, water heaters, air conditioners and refrigerators.
Introduction of new modes of transport, eg: hybrid vehicles, Mass Rapid Transit (MRT), etc. will have a pronounced effect upon future load trends.
Affected by demographic patterns.
Affected by economic patterns.

Industrial customers
These are considered to be base loads that are independent of weather variations.
However, depending on the specific type of industry, they may have specific characteristics.
Eg : Auto ancillary industries, open-cast mining industry.

Others (Municipalities, Boards, Public authorities)

May have seasonal fluctuations depending on specific cases.
Growth trend for this segment is generally stable.

2. (a) Describe the methodology of forecasting in detail. [8 Marks]

Ref : 1) "Power System Reliability, Safety and Management", Balbir Singh Dhillon, Ann Arbor Science, -Page 148-150
2) "Power System Planning", R.L.Sullivan, McGraw-Hill, -Page 24-25

Forecasting is a systematic procedure for quantitatively defining future loads. Depending on time-period of interest, load forecasting may be classified into

a) Short-term
b) Intermediate
c) Long-term

Electric system planning being our basic concern and since planning for the addition of new generation, transmission and distribution facilities must begin 4-10 years ahead of the scheduled events, medium-range / intermediate load forecasting is of more practical interest.

Classification-1
Depending on the specific mathematical method being adopted, load forecasting may also be classified into

a) Extrapolation
b) Correlation
c) A combination of the above

Classification-2
They may also be classified into

a) Deterministic
b) Probabilistic
c) Stochastic

Since Classification-1is of more practical interest, Classification-1 is expanded further.

1. Extrapolation

Extrapolation techniques involve fitting trend curves to basic historical data adjusted to reflect the growth trend. With a trend curve, the forecast is obtained by evaluating the trend curve function at the desired future point. The methods are quite simple, producing reasonably accurate results. These techniques are essentially deterministic extrapolations, since random errors in the data or analytical model are not accounted for. Some of these are listed below.

a) Straight line – linear $y=a + bx$
b) Parabolic $y = a + bx + cx^2$
c) S-Curve $y = a + bx + cx^2 + dx^3$
d) Exponential $y = ce^{dx}$
e) Gompertz $y= Ln^{-1} (a + ce^{dx})$

In all the above curve fitting techniques, the method of least squares is commonly used to determine the coefficients a,b,c & d, as the case may be.

2. Correlation

Correlation techniques of forecasting relate system loads to various demographic and economic factors. The approach is advantageous in forcing the forecaster to understand clearly the relationship between load growth patterns and other measurable factors. The most obvious disadvantage is the need to forecast demographic and economic factors, which can be more difficult than forecasting system load. Typically, factors such as population, employment, building permits, appliance saturation, business indicators, weather data, etc. are used in correlation techniques.

2.(b) Forecast the peak load in 2015 using a linear characteristic. **[12 Marks]**

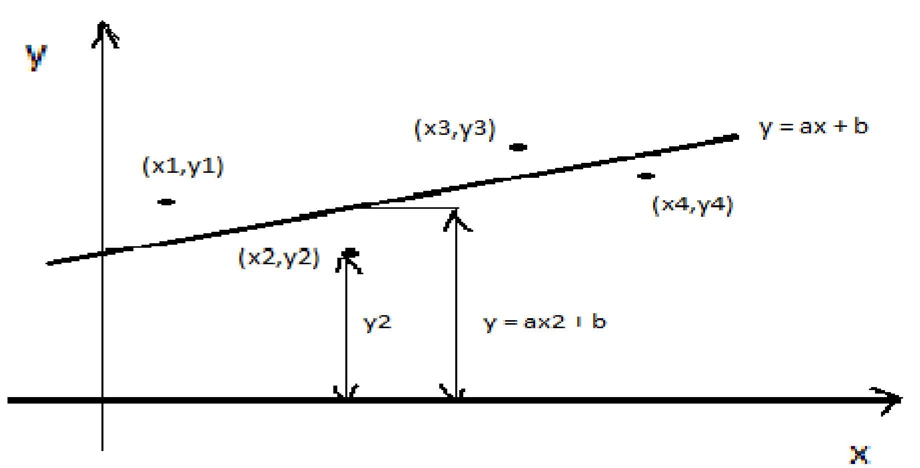
Year	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
Peak Demand (MW)	**71**	**72**	**79**	**81**	**90**	**93**

If we apply linear regression, $Y = aX + b$
Where Y = peak demand (MW)
X = measure of year

We could have taken X to be the year itself. However this will entail more numerical calculation. Hence the given table is modified as follows.

Year	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
X	**1**	**2**	**3**	**4**	**5**	**6**

<u>Theory of linear regression</u>



Corresponding to point (x2,y2), the error in y is equal to e2.

e2 = y2 – y = y2 – (ax2 + b) = (y2 – ax2 – b)

Hence the total error = E

$E = \sum_{i=1}^{n} e_i^2$ where n is the total no. of data sets.

$$E = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

$$\frac{\partial E}{\partial a} = 0$$

$$\sum_{i=1}^{n} 2(y_i - ax_i - b)(-x_i) = 0 \qquad \sum_{i=1}^{n} x_i(y_i - ax_i - b) = 0 \qquad \sum_{i=1}^{n} (x_i y_i) - (ax_i^2) - (bx_i) = 0$$

$$\sum_{i=1}^{n} x_i y_i - a\sum_{i=1}^{n} x_i^2 - b\sum_{i=1}^{n} x_i = 0 \qquad a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$$

Similarly $R = \{1 - [(1 - R_a) * (1 - R_b)]\}$ $\frac{\partial E}{\partial b} = 0$

$$\sum_{i=1}^{n} 2(y_i - ax_i - b)(-1) = 0 \qquad \sum_{i=1}^{n} (y_i - ax_i - b) = 0 \qquad \sum_{i=1}^{n} y_i - a\sum_{i=1}^{n} x_i - nb = 0$$

$$a\sum_{i=1}^{n} x_i + nb = \sum_{i=1}^{n} y_i$$

Hence we have $a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$

and $a\sum_{i=1}^{n} x_i + nb = \sum_{i=1}^{n} y_i$ from which coefficients a and b can be determined.

From the modified table, we have

Year	2009	2010	2011	2012	2013	2014
X	1	2	3	4	5	6

$\sum x_i = 21$; n=6; $\sum y_i = 486$; $\sum x_i^2 = 91$; $\sum x_i y_i = 1784$

Using equations (1) and (2)

21a+6b=486 -(2)

91a+21b=1784 -(1)

$$b = \frac{486 - 21a}{6} = \frac{1784 - 91a}{21}$$

81-3.5a=84.952-4.333a

(4.333-3.5)a-84.952-81

0.833a-3.952

A=4.744 $\therefore$ b=64.396

$\therefore$ y=ax+b=4.744x+64.396

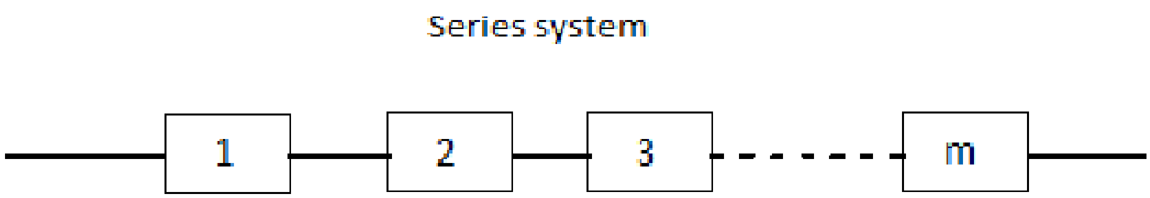
Hence to obtain the demand forecast for the year 2015(x=7)

y=(4.744*7)+64.396=97.6MW

3. (a) How will you determine the reliability in series and parallel systems? [10 Marks]

Ref : 1) "Power System Reliability, Safety and Management", Balbir Singh Dhillon, Ann Arbor Science, -Page -Page 57, 62
2) "Reliability Modeling in Electric Power Systems"- J.Endrenyi, Wiley-39, 41

Series network



This is the simplest reliability configuration encountered. A series system is shown below. All the components must function successfully for system success. In other words, if any system component fails, the series system fails.

R_s = Network reliability

R_s = Prob. (A1.A2.A3………Am)

Where Aj = Event that the jth component of the series system is functioning normally (for j=1,2,3,…….,m)

Prob. (A1.A2.A3………Am) represents the probability of success events(.) or (intersection of) success events(.)

m= no. of components

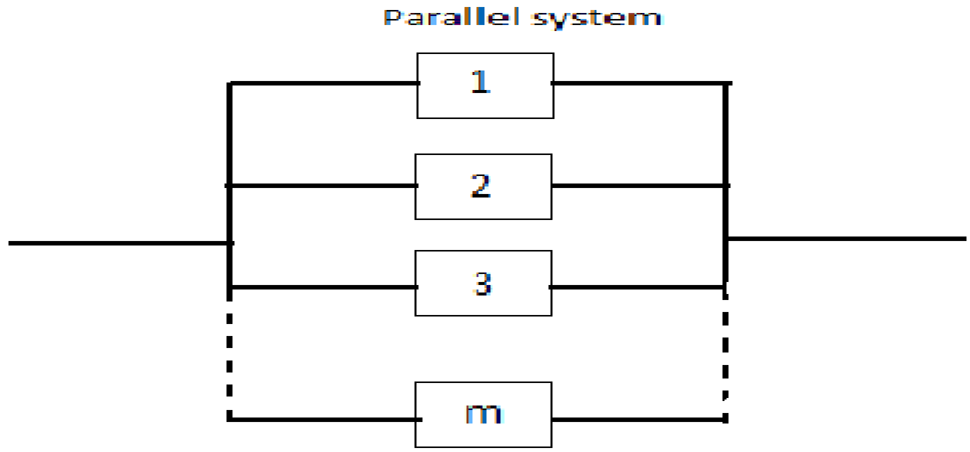
For statistically independent events, the above equation reduces to

R_s = Prob.(A1).Prob.(A2). Prob.(A3)……..Prob.(Am).

For P(Aj) = Rj , R_s= R1.R2.R3…………Rm, $R_s = \prod$ Rj

Where Rj = jth component reliability

Parallel network



This is a well known configuration used to improve system reliability. All the units of the network are assumed to be active. At least one parallel unit of the network is required for system success. In other words, the system will be successful if at least one network unit is operating successfully. The failure probability of the network shown above is given by

R_p = Network reliability

F_p = Network unreliability = 1- R_p

F_p = Prob.($\bar{A}_1.\bar{A}_2.\bar{A}_3..........\bar{A}_m$)

Where $\bar{A}_j$ = event that the ith component of the parallel system is not functioning (for i =1,2,3,….m)

m= no. of components

Prob. ($\bar{A}_1.\bar{A}_2.\bar{A}_3..........\bar{A}_m$) represents the probability of failure events(.) or (intersection of) failure events(.)

In the case of statistically independent events, the above equation reduces to

$F_p = \Pr ob(\bar{A}_1).\Pr ob.(\bar{A}_2).\Pr ob.(\bar{A}_3).......\Pr ob.(\bar{A}_m)$

$F_p = \prod_{i=1}^{m} F_i$ where F_i = ith component failure probability

Since R_p + F_p = 1, R_p = 1- F_p

R_p = 1- $\prod_{i=1}^{m} F_i$ = $\{1-\prod_{i=1}^{m}(1-R_i)\}$

Hence R_p = 1- $(1-R)^m$ = 1-$\{(1-R_1)(1-R_2)(1-R_3)..........(1-R_m)\}$

Where $R_1, R_2, ...R_m$ represent the individual component reliabilities.

When all the components are identical, $R_1 = R_2 = ... = R_m = R$

Then R_p = 1-$(1-R)^m$

3(b) Explain the probability models for generating units and loads. [10 Marks]

Ref: 1) Reliability evaluation of power systems – Billinton & Allan, Plenum Press-Pg.20

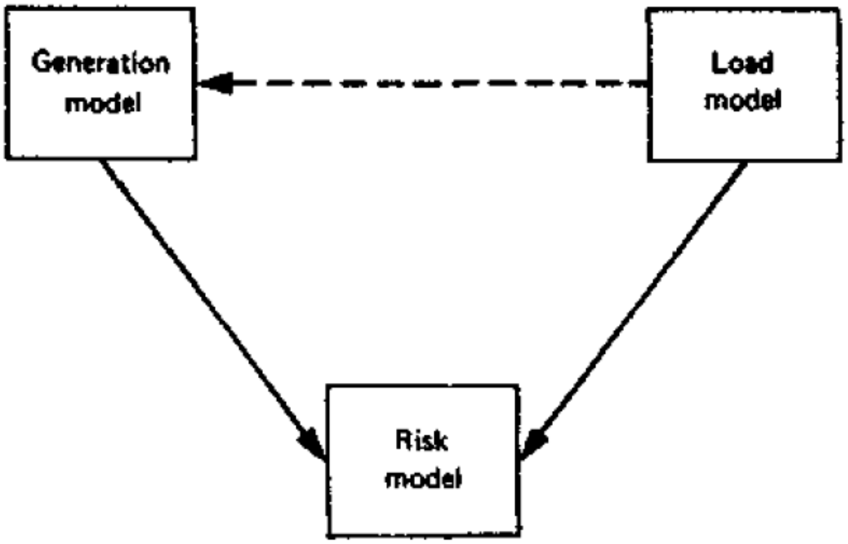


Fig. **2.1** Conceptual tasks in generating capacity reliability **evaluation**

The basic approach to evaluating the adequacy of a particular generation configuration is fundamentally the same for any technique. It consists of three parts as shown in Fig. 2.1.

The generation and load models shown in Fig. 2.1 are combined (convolved) to form the appropriate risk model. The system representation in a conventional study is shown in Fig. 2.2.

Fig. 2.2 Conventional system model

2,2 The generation System model

2.2.1 Generating unit unavailability

The basic generating unit parameter used in static capacity evaluation is the probability of finding the unit on forced outage at some distant time in the future. This probability was defined in Engineering Systems as the unit unavailability, and historically in power system applications it is known as the unit forced outage rate (FOR). It is not a rate in modern reliability terms as it is the ratio of two time values. As shown in Chapter 9 of Engineering Systems,

$$\text{Unavailability (FOR)} = U = \frac{\lambda}{\lambda+\mu} = \frac{r}{m+r} = \frac{r}{T} = \frac{f}{\mu}$$

$$= \frac{\Sigma[\text{down time}]}{\Sigma[\text{down time}] + \Sigma[\text{up time}]} \qquad 2.1(a)$$

$$\text{Availability} = A = \frac{\mu}{\lambda+\mu} = \frac{m}{m+r} = \frac{m}{T} = \frac{f}{\lambda}$$

$$= \frac{\Sigma[\text{up time}]}{\Sigma[\text{down time}] + \Sigma[\text{up time}]} \qquad 2.1(b)$$

where X = expected failure rate
u = expected repair rate
m = mean time to failure = MTTF = I/A.
r = mean time to repair = MTTR = 1/u
m + r= mean time between failures = MTBF = l/f
/= cycle frequency = l/T
T= cycle time = l/f.
The concepts of availability and unavailability as illustrated in Equations 2.1 (a) and (b) are associated with the simple two-state model shown in Fig. 2.3(a). This model is directly applicable to a base load generating unit which is either operating or forced out of service. Scheduled outages must be considered separately as shown later in this chapter.
In the case of generating equipment with relatively long operating cycles, the unavailability (FOR) is an adequate estimator of the probability that the unit under similar conditions will not be available for service in the future. The formula does not, however, provide an adequate estimate when the demand cycle, as in the case of a peaking or intermittent operating unit, is relatively short. In addition to this, the most critical period in the operation of a unit is the start-up period, and in comparison with a base load unit, a peaking unit will have fewer operating hours and many more start-ups and shut-downs. These aspects must also be included in arriving at an estimate of unit unavailabilities at some time in the future.

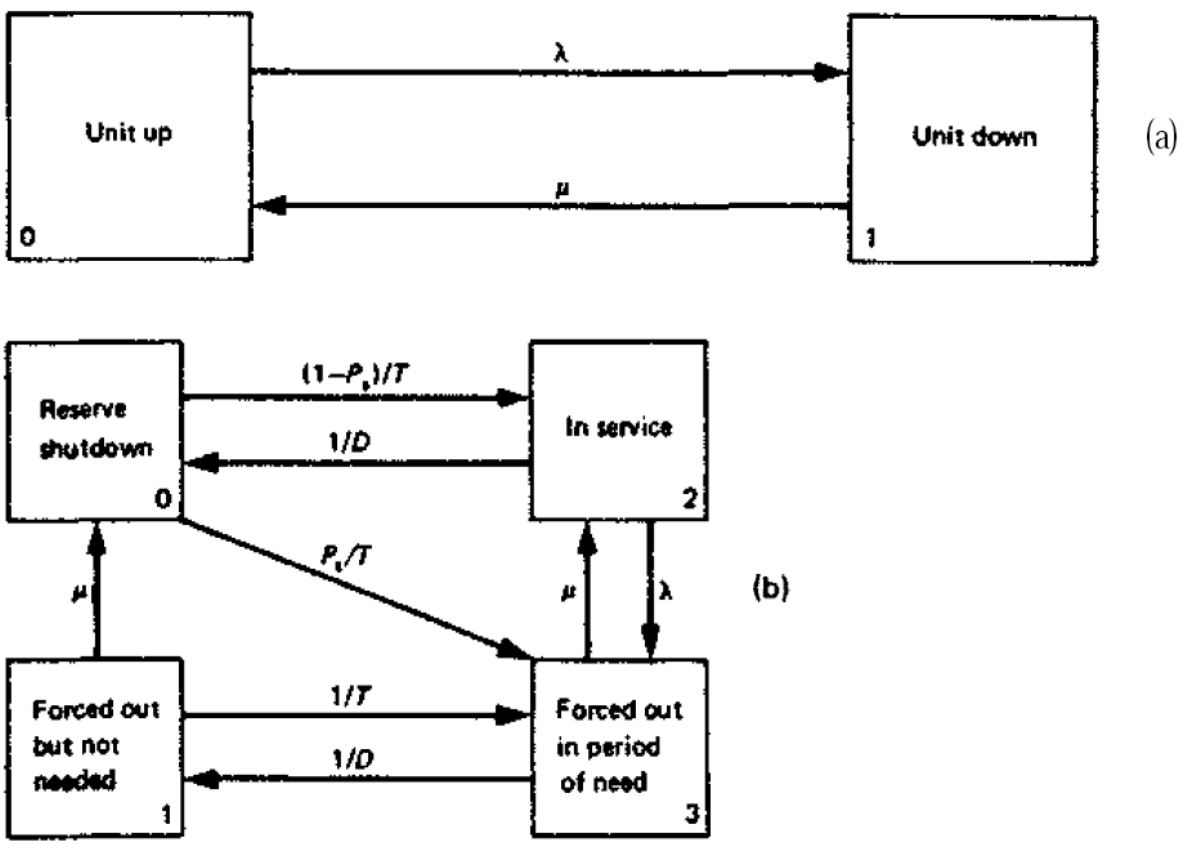


Fig. 2.3 (a) Two-state model for a base load unit
(b) Four-state model for planning studies
T Average reserve shut-down time between periods of need
D Average in-service time per occasion of demand
P_s Probability of starting failure

The difference between Figs 2.3(a) and 2.3(b) is in the inclusion of the 'reserve shutdown' and 'forced out but not needed' states in Fig. 2.3(b). In the four-state model, the 'two-state' model is represented by States 2 and 3 and the two additional states are included to model the effect of the relatively short duty cycle. The failure to start condition is represented by the transition rate from State 0 to State 3.

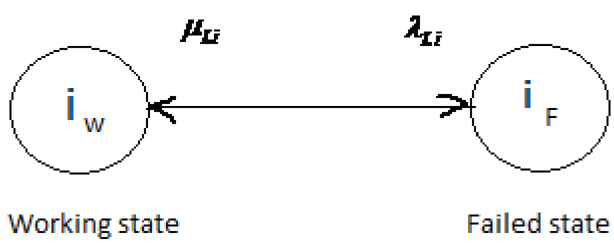
Load model

Ref : "Reliability Modeling in Electric Power Systems"- J.Endrenyi, Wiley JE – Pg-194

The bus loads change continuously and so does the resultant system load. As a consequence, a given system state may represent success for one load condition and system failure for another. A line outage during which some other line becomes overloaded at high-load periods may not have the same effect when the loads are low. In general, a probability can be assigned to a system which is defined as q_i = P[System failure|system is in state i]

Thus q_i indicates the proportion of time for which state I is a system failure state. If the state I is split into two sub-states, one successful and the other failed (Fig. 3.1), q_i can be expressed in terms of the transition rates λ_{Li} and μ_{Li} as $q_i = \frac{\lambda_{Li}}{\lambda_{Li}+\mu_{Li}}$

Fig.3.1



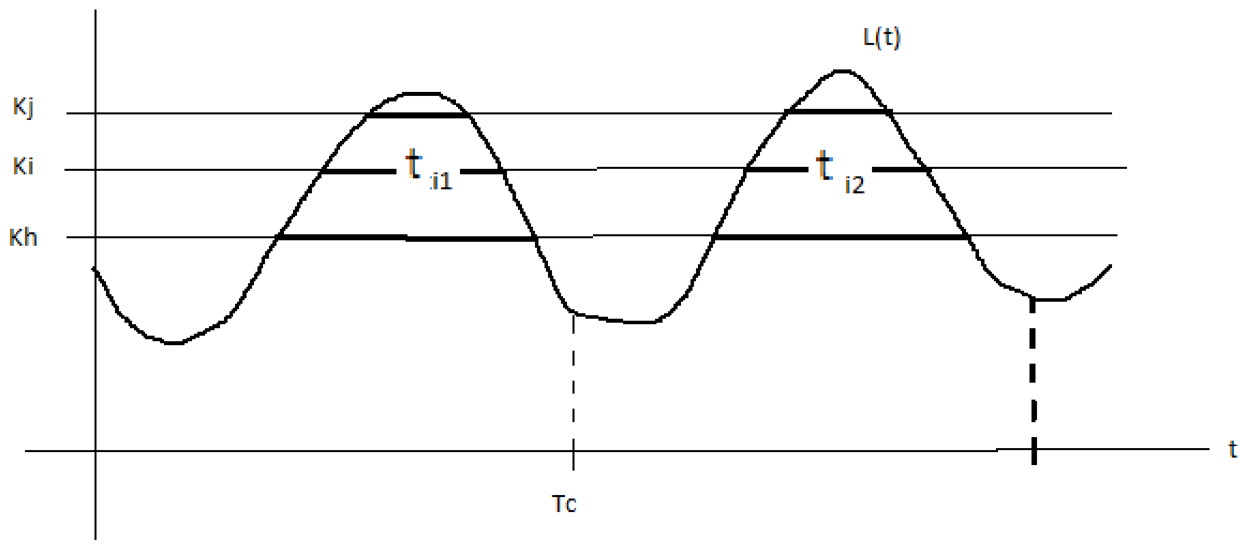
The system failure probability P_F is now expressed as $P_F = \sum_{i=n} p_i q_i$

q_i, λ_{Li} and μ_{Li} need to be determined for the various states. These values depend on the load model employed. Two such load models are of importance.

In the first model, the bus loads are assumed to be independent. In the second, they are fully correlated (changing simultaneously and in the same proportion).

The second model is shown in Fig.3.2. Here, the load curve is represented.

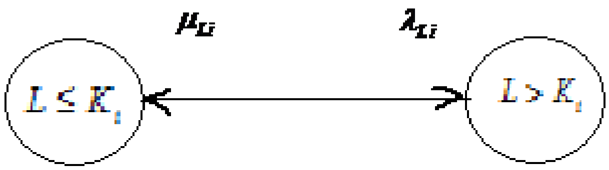
Fig.3.2 Load representation for correlated bus loads



The curve L(t) is the load curve, given in p.u values. The cycle time Tc represents one day. For any state I, one can define a load level Ki, such that if the system load exceeds it, the system is failed in i. However, if L < Ki, the system is working in i. If the mean of the durations $\mu_{Li} = \frac{1}{T_i} t_{i1}, t_{i2}$, etc. for which L > Ki, the probability q_i can be expressed by $q_i = P[L > K_i] = \frac{T_i}{T_C}$

The diagram in Fig.3.1 can now be converted into a simple load model for state I, as shown in Fig.3.3.

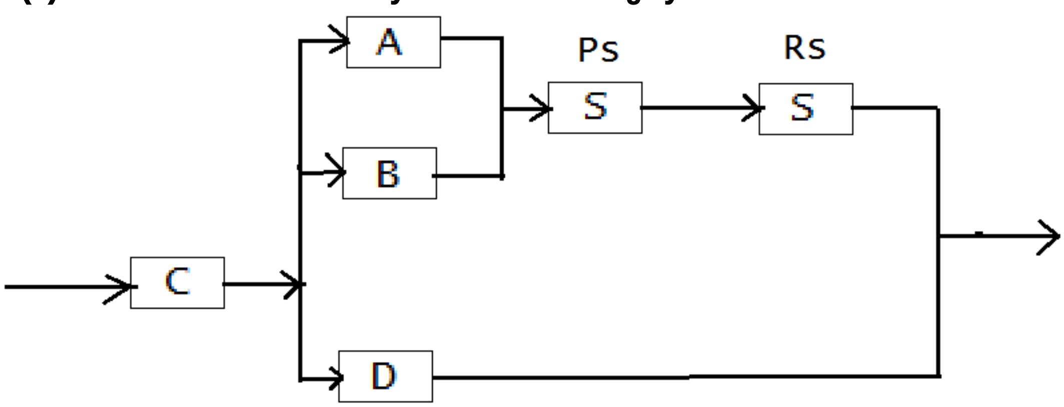
Fig.3.3 Load model for state i



The rates λ_{Li} and μ_{Li} are given by

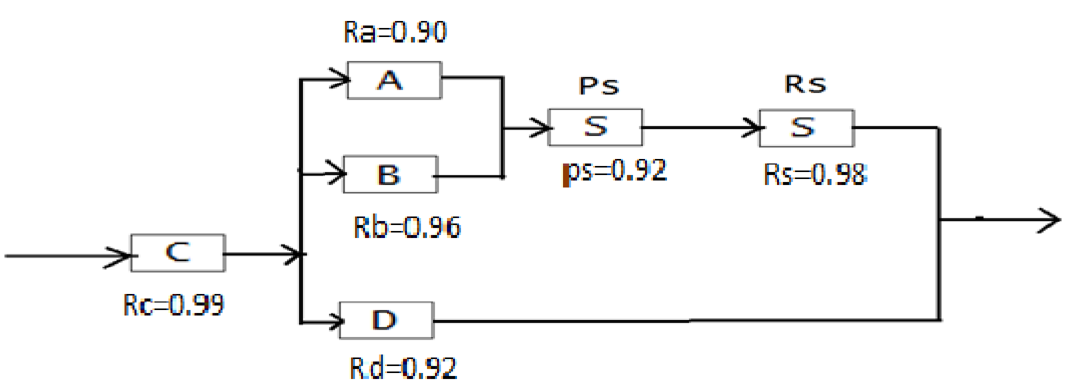
$$\lambda_{Li} = \frac{1}{T_c - T_i} \text{ and } \mu_{Li} = \frac{1}{T_i}$$

4. (a) Evaluate the reliability of the following system. **[10 Marks]**

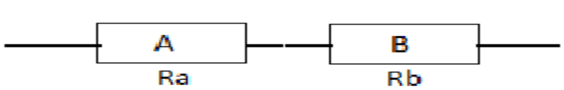


Ra=0.9 **Rb=0.96** **Rc=0.99** **Rd=0.8** **Ps=0.92** **Rs=0.98**

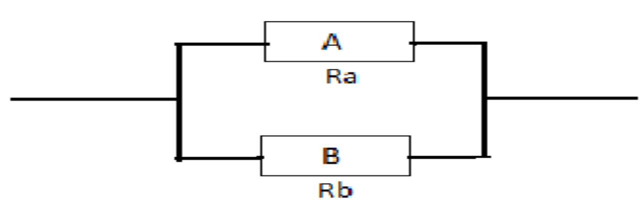
We assume that the given values indicate the reliabilities of the individual units.



For a series system

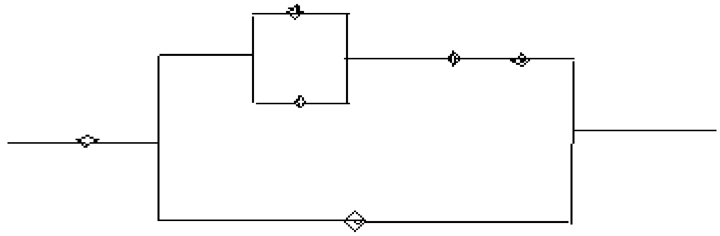


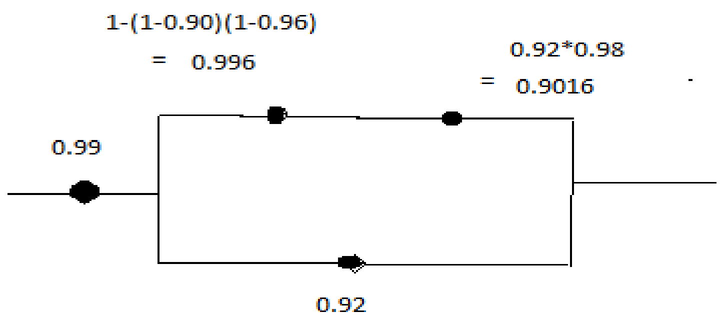
For a parallel system

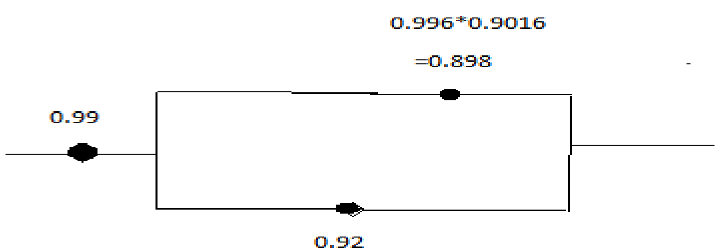


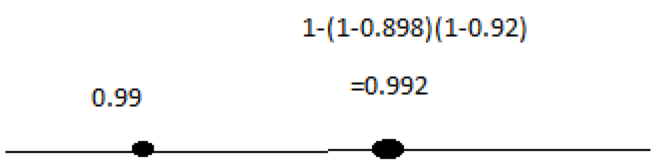
$R = R_a * R_b$ $\qquad\qquad$ $R = \{1 - [(1 - R_a) * (1 - R_b)]\}$

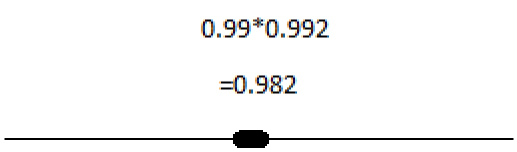
Hence for the given system

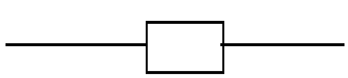












R=0.982

4 (b) Assume a generating system consisting of the following machines with their associated outage rates.

MW	Outage rate
10	0.02
10	0.02
10	0.02
10	0.02
5	0.02

Compute the probability outage table for the first four units. [10 Marks]

For a general case, we consider all the units = (4*10 MW) + (1*5 MW)

Considering (4 x 10 MW)

Table-1

Capacity in	Probability
0	$4C_0 * 0.98^0 * 0.02^4 = 1.6000E-7$
10	$4C_1 * 0.98^1 * 0.02^3 = 3.136E-5$
20	$4C_2 * 0.98^2 * 0.02^2 = 2.305E-3$
30	$4C_3 * 0.98^3 * 0.02^1 = 0.0753$
40	$4C_4 * 0.98^4 * 0.02^0 = 0.9224$

Considering (1 x 5 MW)

Table-2

Capacity in	Probability
0	0.0200
10	0.9800

Combining the (4*10 MW) and the (1*5 MW) units

Table-3

Capacity In	Probability	Prob. Figure
0+0=0	1.6700 E -07 * 0.02	3.2000 E -09
0+5=5	1.6700 E -07 * 0.98	1.5680 E -07
10+0=10	3.1360 E -05 * 0.02	6.2720 E -07
10+5=15	3.1360 E -05 * 0.98	3.0734 E -05
20+0=20	2.3050 E -03 * 0.02	4.6100 E -05
20+5=25	2.3050 E -03 * 0.98	2.2589 E -03

30+0=30	0.0753	* 0.02	1.5060 E -03
30+5=35	0.0753	* 0.98	0.0738
40+0=40	0.9224	* 0.02	0.0184
40+5=45	0.9224	* 0.98	0.9039

Reversing the table, we have

Table-4

Capacity In	**Outage**	**Prob. Figure**	**Cumulative Probability**
45	0	0.9039	1.0000
40	5	0.0184	0.0960
35	10	0.0738	0.0776
30	15	1.5060 E -03	3.8430 E -03
25	20	2.2589 E -03	2.3365 E -03
20	25	4.6100 E -05	7.7620 E -05
15	30	3.0734 E -05	3.1520 E -05
10	35	6.2720 E -07	7.8720 E -07
5	40	1.5680 E -07	1.6000 E -07
0	45	3.2000 E -09	3.2000 E -09

This is the required capacity outage table. **However, the question paper asks for only the table relevant to the first four units.**

Table-5 (Same as Table-1)

Capacity in	**Probability**
0	1.6000 E -07
10	3.1360 E -05
20	2.3050 E -03
30	0.0753
40	0.9244

Reversing the table rows, we have

Table-6

Capacity in	Outage	Probability	Cumulative probability
40	0	0.9244	1.0000
30	10	0.0753	0.0776
20	20	2.3050 E -03	2.3355 E -03
10	30	3.1360 E -05	3.1520 E -05
0	40	1.6000 E -07	1.6000 E -07

This is the required outage table.

5. (a) Describe how transmission system reliability can be analyzed. [10 Marks]

Ref : 1) "Power System Planning", R.L.Sullivan, McGraw-Hill, Pg--237 –1st part
2) "Power System Reliability, Safety and Management", Balbir Singh Dhillon, Ann Arbor Science, -Page 221, 223-2nd part

The probabilistic approach to reliability analysis is a powerful tool, to be used with discretion
The probabilistic approach to reliability analysis is a powerful tool, to be used with discretion
It does not replace detailed analysis such as AC power flows. Probabilistic methods are used to seek out and identify transmission bottle-necks and trouble spots. Such methods provide additional insight, reduce computational time and most importantly, allow the planner to focus attention on areas most likely to create transmission problems.

The determination of the transmission system reliability relies more on the probabilistic approach than a deterministic approach. It is commonly expressed in terms of two indices.

1) LOLP (Loss of load probability)
2) e(DNS) – Expected value of (Demand Not served)

These two methods enable the system planner to describe the capacity of each element (including generating units) in the system with a probability distribution function. The LOLP and the e(DNS) for each element is indicative of its relative role in determining the reliability of the system.
To illustrate, if the reliability of a given transmission expansion plan is not sufficiently high, those elements most responsible for poor reliability can be identified. Transmission system reliability is generally not carried out in isolation; it often includes generating capacity reliability analysis as well. This is because, the outage characteristics of generating units clearly influence the loading of the system and hence its reliability.

Basic philosophy-2 state model

Each transmission element can be described adequately by two-state models and associated FOR's (Forced outage rates). The treatment that follows adopts this two-state model throughout. *In addition, to simulate real-life conditions, without having to go in for higher-order element models, we adopt a fluctuating environment as well. This is dealt with separately.*

Given that each element Vm, m=1,2,….,n in the system under study can reside in the "0" state with probability q_m, in which it has no capacity and is hence out of service, or the "I" state, with probability p_m, in which it has capacity c_m and is in service. Thus there will be 2^n distinct capacity states X_i, i=1,2,…., 2^n. For instance, a three-bus system has five elements; two generators and three lines. Therefore, the system can reside in any of 2^5 = 32 different capacity states X_i. **Assumption : De-rated states are not considered. This will compound the problem.**
Obviously the upper and lower limiting states denoted by

$\overline{X}$ = (1,1,1,1,1) and $\underline{X}$ = (0,0,0,0,0)

Associated with each of the 2^n states is a probability f(X_i) that it will occur. For example, the probability f(X_i) that the three-bus system will reside in the upper limiting state is $f(\overline{X}) = \prod_{m=1}^{n} p_m = p_1 p_2 p_3 p_4 p_5$

Similarly, the probability that the system will reside in the lower limiting state would be

$$f(\underline{X}) = \prod_{m=1}^{n} q_m = q_1 q_2 q_3 q_4 q_5$$

In general, the probability that the system will reside in any state Xi =(V1,V2,…..Vm) is

$$f(X_i) = \prod_{m=1}^{n} f(V_m)$$ where f(Vm)= p_m if Vm=1

where f(Vm)= q_m if Vm=0

To summarize, we must decompose the set Xi, i=1,2,…., 2^n of system capacity states into states that are acceptable and states that are unacceptable. Unacceptable states are system capacity states Xi for which the load L, cannot be satisfied, either because of insufficient generation capacity or because of insufficient transmission capacity. Assuming that we can single out the unacceptable states, the system LOLP is defined by

$$LOLP = \sum f(X_i)$$ X_i=all unacceptable states

Similarly for each unacceptable state X_i, which has a probability of occurrence $f(X_i)$, the amount of load

$$\lim_{s \to 0} R_c(s) = MTTF = \frac{\lambda_{sl} + \alpha_s + \alpha_n}{(\lambda + \alpha_n)(\lambda_{sl} + \alpha_s) - \alpha_n \alpha_s}$$

served is $$MTTF = \frac{\lambda_{sl} + \alpha_s + \alpha_n}{\lambda_{nl}\lambda_{sl} + \alpha_n \lambda_{sl} + \lambda_{nl}\alpha_s}$$ g_i < L, then the e(DNS) would be the sum of the

$$\lambda = \frac{1}{MTTF} = \frac{\lambda_{nl}\lambda_{sl} + \alpha_n \lambda_{sl} + \lambda_{nl}\alpha_s}{\lambda_{sl} + \alpha_s + \alpha_n}$$

products of demand not served and the probability that the associated state occurred.

e(DNS) = Σ f(Xi) (L- g_i) X_i=all unacceptable states

Note that (L- g_i) is the amount of demand not served because the system capacity state X_i is unacceptable.

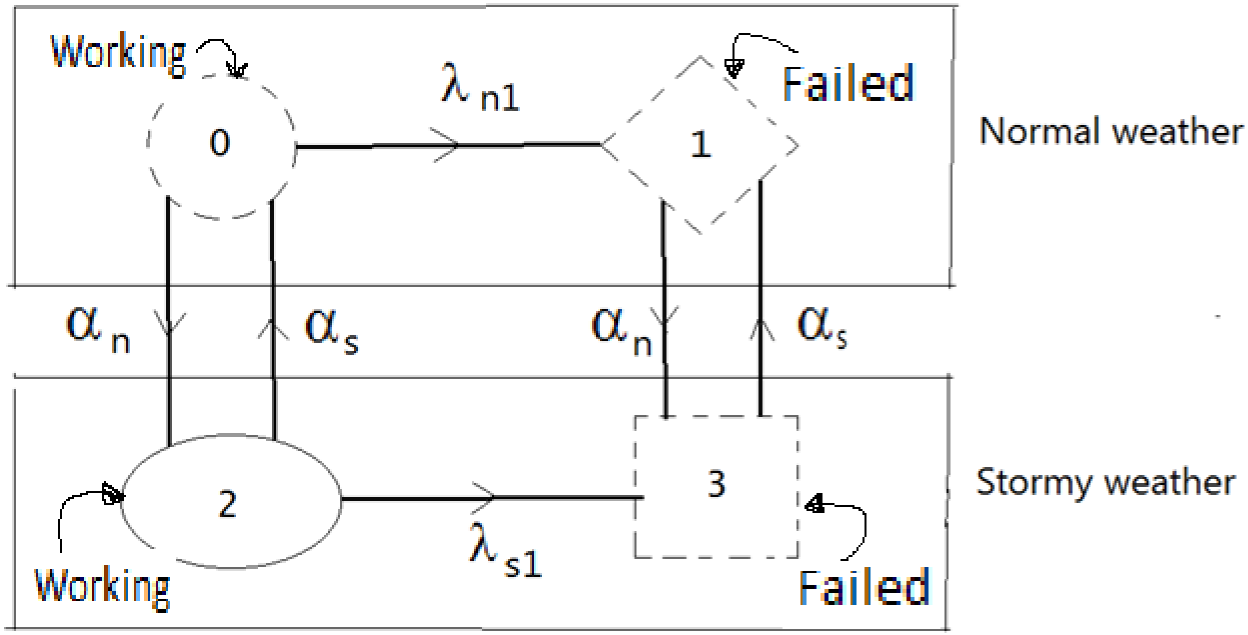
DNS = (L- g_i)

Basic philosophy- fluctuating environment

The transmission system is subject to a wide variety of weather fluctuations. Hence its reliability evaluation depends on the analysis of a component (in a transmission system or otherwise) in a fluctuating environment.

A transmission component installed outdoors is normally subject to a two-state alternating environment(i.e., normal and stormy weather). Due to this, component failure and repair rates(each normally assumed constant) vary from one weather state to another. To predict component reliability more accurately, the state space diagram of a Markov model is shown in Fig.5.

Fig.5 State transition diagram



λ_{nl} = Constant failure rate of the component operating in the normal environment

λ_{sl} = Constant failure rate of the component operating in the stormy environment

α_n = Constant rate of the weather going from normal to stormy state

α_s = Constant rate of the weather going from stormy to normal state

$P_i(t)$ = ith state probability at time i=0,1,2,….

s = Laplace transform variable

The system of differential equations associated with the above figure is

$$\frac{dP_0(t)}{dt}+(\lambda_{nl}+\alpha_n)P_0(t)=P_2(t)\alpha_s$$ - Eqn. 5b1

$$\frac{dP_1(t)}{dt}+\alpha_n P_1(t)=P_0(t)\lambda_{nl}+P_3(t)\alpha_s$$ - Eqn. 5b2

$$\frac{dP_2(t)}{dt}+(\lambda_{sl}+\alpha_s)P_2(t)=P_0(t)\alpha_n$$ - Eqn. 5b3

$$\frac{dP_3(t)}{dt} + \alpha_s P_3(t) = P_1(t)\alpha_n + P_2(t)\alpha_{sl}$$ - Eqn. 5b4

At t=0, $P_o(0) = 1$ and $P_1(0) = P_2(0) = P_3(0) = 0$

Solving the above four equations using Laplace transforms, we get the state probability equations,

$$P_0(s) = \frac{(s + \lambda_{sl} + \alpha_s)}{A}$$

Where $A = [(s + \lambda_{nl} + \alpha_n)(s + \lambda_{sl} + \alpha_s) - \alpha_n \alpha_s]$

$$P_1(s) = \{[(s + \lambda_s + \alpha_s)\lambda_{nl} + \frac{\alpha_s \lambda_{nl} \lambda_{sl}}{s + \alpha_s}][s+\alpha_n - \frac{\alpha_n \alpha_s}{s+\alpha_s}]^{-1}\} * \frac{1}{A}$$

$$P_2(s) = \frac{\alpha_n}{A}$$

$$P_3(s) = \frac{P_1(s)\alpha_n}{s + \alpha_s} + \frac{\alpha_n \lambda_{sl}}{A(s + \alpha_s)}$$

Laplace transforms of component reliability $R_c(t)$ in both weather conditions are

$$R_c(s) = P_0(s) + P_2(s) = \frac{(s + \lambda_{sl} + \alpha_s) + \alpha_n}{A}$$

The component mean time to failure (MTTF) is obtained by letting s-> 0 in the above equation.

$$\lim_{s \to 0} R_c(s) = MTTF = \frac{\lambda_{sl} + \alpha_s + \alpha_n}{(\lambda + \alpha_n)(\lambda_{sl} + \alpha_s) - \alpha_n \alpha_s}$$

$$MTTF = \frac{\lambda_{sl} + \alpha_s + \alpha_n}{\lambda_{nl}\lambda_{sl} + \alpha_n \lambda_{sl} + \lambda_{nl}\alpha_s}$$

Using the above equation, the component mean failure rate under both weather conditions is given by

$$\lambda = \frac{1}{MTTF} = \frac{\lambda_{nl}\lambda_{sl} + \alpha_n \lambda_{sl} + \lambda_{nl}\alpha_s}{\lambda_{sl} + \alpha_s + \alpha_n}$$

5.(b) Describe with a typical example the frequency and duration method. [10 Marks]

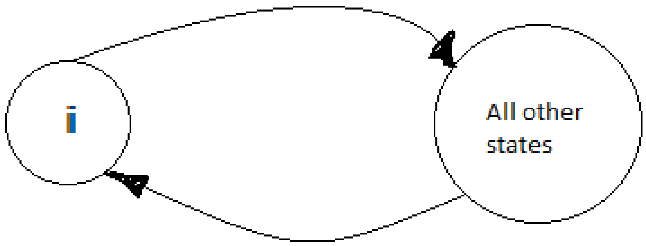
Ref : 1) "Reliability Modeling in Electric Power Systems"- J.Endrenyi, Wiley, Pg-53 [taken from Q.7(b)]-Theory

2) "Power System Reliability, Safety and Management", Balbir Singh Dhillon, Ann Arbor Science, -Page 200-Example

The frequency of encountering state i, f_i, is defined as the expected number of stays in (or arrivals into, or departures from) i per unit time, computed over a long period. By this definition, the concept of frequency is associated with the long term behavior of the process describing the system. The mean duration of the stays in state i must also be computed over a long period of time.

In order to relate the frequency, probability, and mean duration of a given system state, the history of the system will be regarded as consisting of two alternating periods, the stays in i and the stays outside i. Thus the system is represented by a two-state process whose state-space diagram is shown in Fig.7.1. Let the mean duration of the stays in state i be T_i and that of the stays outside I, be $T_i^{'}$.

Fig.- 7.1 Two-state process



The mean cycle time , $T_{ci} = T_i + T_i^{'}$

From the definition of the state frequency, it follows that in the long run, f_i equals the reciprocal of the mean cycle time.

$$f_i = \frac{1}{T_{ci}}$$

Multiplying the above eqn. by T_i, we have $p_i = \dfrac{T_i}{T_{ci}}$

When we analyze the long-term behavior of a component, its probability of being in the up-state is the ratio of the mean up-time to the sum of the mean up and down-times (proportion of the time that the component is working). Similarly its probability of being in the down-state is the ratio of the mean down-time to the sum of the mean up and down-times (proportion of the time that the component is not working).

$$p_i = \frac{T_i}{T_{ci}}$$

Hence $f_i T_i = p_i$ where p_i represents the probability of being in state i.

This is a fundamental equation which provides the relation between the three parameters.

Next, the frequencies f_i, mean durations T_i and the transition rates in the system will be related. To begin with, the concept of the frequency of transfer from state i to state j is introduced. This frequency f_{ij} is introduced. This frequency f_{ij} is defined as the expected number of direct transfers from i to j per unit time.

We know that the intensity of transition from state i to state j, $q_{ij}(t)$ is defined as

$$q_{ij}(t) = \underset{\vartriangle t \to 0}{Lim} \frac{1}{\vartriangle t} P[X(t + \vartriangle t) = j \mid X(t) = i])$$

$$\sum_{i=1}^{n} x_i y_i - b \sum_{i=1}^{n} x_i = 0$$

$$a \sum_{i=1}^{n} x_i^{2} + b \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$$

Where X(t) is the random variable representing the system state at time t, and similarly for $X(t + \Delta t)$

$$f_{ij} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P[(X(t+\Delta t) = j) \cap (x(t) = i)]$$

$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} P[X(t+\Delta t) = j \mid x(t) = i] P[X(t) = i]$$

$$= \lambda_{ij} p_i \quad \text{where } \lambda_{ij} \text{ is the transition rate}$$

Thus, λ_{ij} is essentially a conditional frequency, the condition being that the system resides in *i*. Now, from the definitions of f_i and f_{ij} it follows, that:

$$f_i = \sum_{j \neq i} f_{ij}.$$

Hence, $f_i = p_i \sum_{j \neq i} \lambda_{ij}$.

We already know that $f_i T_i = p_i$

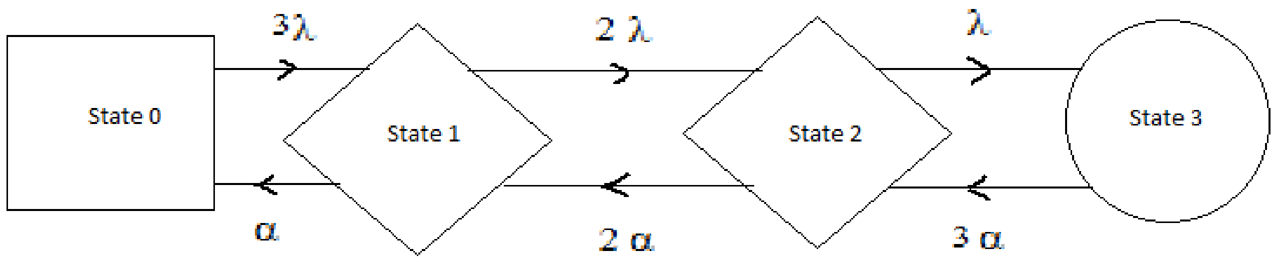
Hence, $T_i = \frac{1}{\sum_{j \neq i} \lambda_{ij}}$

Thus it can be inferred that the mean duration of the stays in any given state equals the reciprocal of the total rate of departures from that state. All the state indices can now be computed from the transition rates that define a given system.

Eg:

Three independent, identical, repairable, active 500 MW generators form a parallel system. The constant failure and repair rates for each generator are λ = 0.005 Failures/hr. and μ = 0.01 repairs/hr. respectively. For 0 Mw power generation, calculate the system steady-state probability, frequency of encountering mean duration and mean cycle time. The system state space diagram is shown in Fig.5.2

Fig.5.2



State 0 : Three generators operating

State 1 : Two generators operating; one failed

State 2 : One generator operating; two failed

State 3 : All generators failed

By using the Markov technique, the system of differential equations associated with Fig.5.2 is

$Q_s = \{P_g(1,2) + P_c(1) - P_g(1,2).P_c(1)\}.R_{L3} + \{P_g(1,2) + P_c(2) - P_g(1,2).P_c(2)\}.Q_{L3}$ -Eqn.1

$$\frac{dP_1(t)}{dt} + (2\lambda + \alpha)P_1(t) = P_0(t).3\alpha + P_2(t).2\alpha \quad \text{-Eqn.2}$$

$$\frac{dP_2(t)}{dt} + (\lambda + 2\alpha)P_2(t) = P_3(t).3\alpha + P_1(t).2\lambda \quad \text{-Eqn.3}$$

$$\frac{dP_3(t)}{dt} + 3\alpha P_3(t) = P_2(t).\lambda \quad \text{-Eqn.4}$$

Where $P_i(t) = i_{th}$ state probability at time t, for i=0,1,2,3
At t=0, $P_0(0) = 1$ and $P_1(0) = P_2(0) = P_3(0) = 0$

For large t, the solution equations are

$$P_0 = \frac{\alpha^3}{(\lambda + \alpha)^3} \quad \text{-Eqn.5}$$

$$P_1 = \frac{3\alpha^2\lambda}{(\lambda + \alpha)^3} \quad \text{-Eqn.6}$$

$$P_2 = \frac{3\alpha\lambda^2}{(\lambda + \alpha)^3} \quad \text{-Eqn.7}$$

$$P_3 = \frac{\lambda^3}{(\lambda + \alpha)^3} \quad \text{-Eqn.8}$$

From Eqn.8, the probability of)-MW power output is $P_3 = \frac{\lambda^3}{(\lambda + \alpha)^3}$ = 0.0370

The frequency of encountering State-3, $F_3 = 3\alpha.P_3 = 3\alpha.\frac{\lambda^3}{(\lambda + \alpha)^3}$ = 0.00111/Hr. -Eqn.9

The mean cycle time between encountering a Markov state is $T = F^{-1}$
Hence $T_3 = \frac{1}{F_3}$ =900.9009 Hrs.

The mean residence time in a Markov state = $T_r = \frac{1}{Sumofoutgoingtransitionrates)}$

Hence $T_{r3} = \frac{1}{3\alpha}$ = Mean residence time in State 3 = 33.33 Hr.

6. (a) Give a short note on two plant style load system. **[8 Marks]**

Ref : 1) "Power System Reliability Evaluation"-Roy Billinton, Gordon & Breach Science Publishers-Pg-220

Two plant style load systems are referred in the context of composite / bulk power system reliability. Here, the net effect of both the generating and transmission system is taken into consideration while arriving at the system reliability. This is achieved by using suitable reliability indices.

Two plant style load systems are generally of two types.

- Two plant – single load system configuration
- Two plant– two load system configuration

Fig.6a-1 Two-plant-Single load system configuration

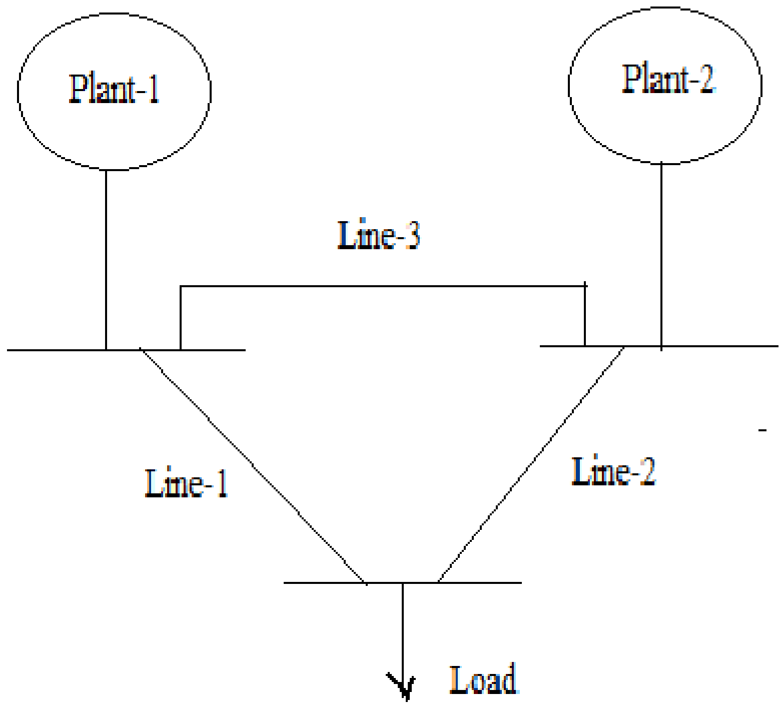
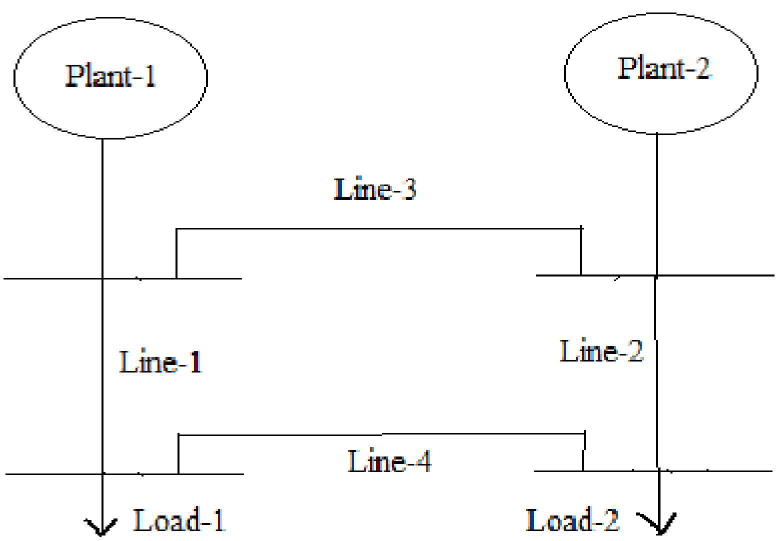


Fig.6a- 2 Two-plant-Single load system configuration



Only the first configuration is dealt with in this solution.

$$Q_s = Q_s(L_1 - In).R_{L1} + Q_s(L_1 - Out).Q_{L1}$$

For L1 In

$$Q_s = Q_s(L_2 - In).R_{L2} + Q_s(L_2 - Out).Q_{L2}$$

For L1 In and For L2 In

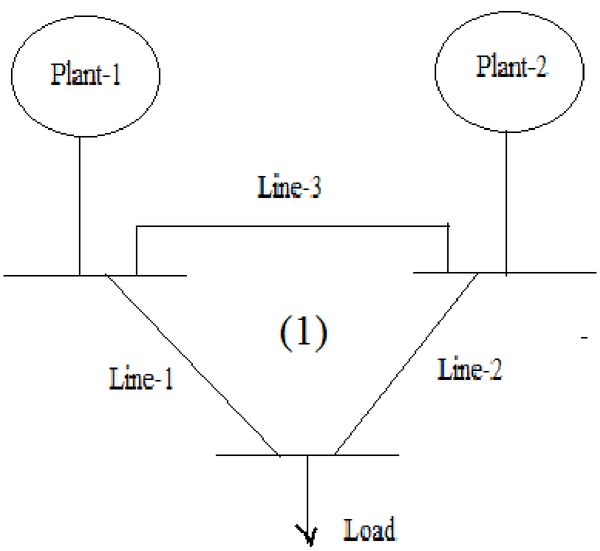
$$Q_s = P_g(2) + P_c(6) - P_g(2).P_c(6)$$

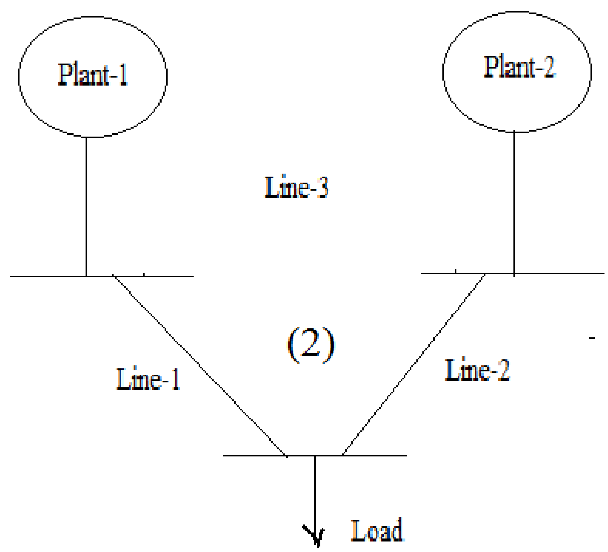
$$Q_s = \{P_g(1,2) + P_c(1) - P_g(1,2).P_c(1)\}.R_{L3} + \{P_g(1,2) + P_c(2) - P_g(1,2).P_c(2)\}.Q_{L3}$$

Where

$P_g(1,2)$ =probability of load curtailment for both generating plants.

$P_c(1)$ and $P_c(2)$ are the curtailment probabilities for the transmission configurations shown below.





For most practical cases, $P_c(1) = P_c(0)$ and therefore $Q_s = P_g(1,2)$

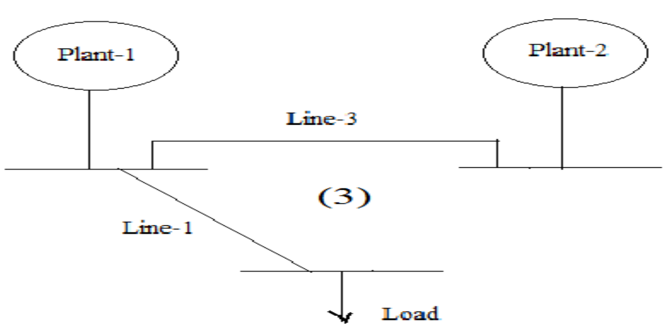
For L1 In and L2 Out

$$Q_s = Q_s(L_3 - In).R_{L3} + Q_s(L_3 - Out).Q_{L3}$$

For L1 In and L2 Out and L3 In

$$Q_s = \{P_g(1,2) + P_c(3) - P_g(1,2).P_c(3)\}$$

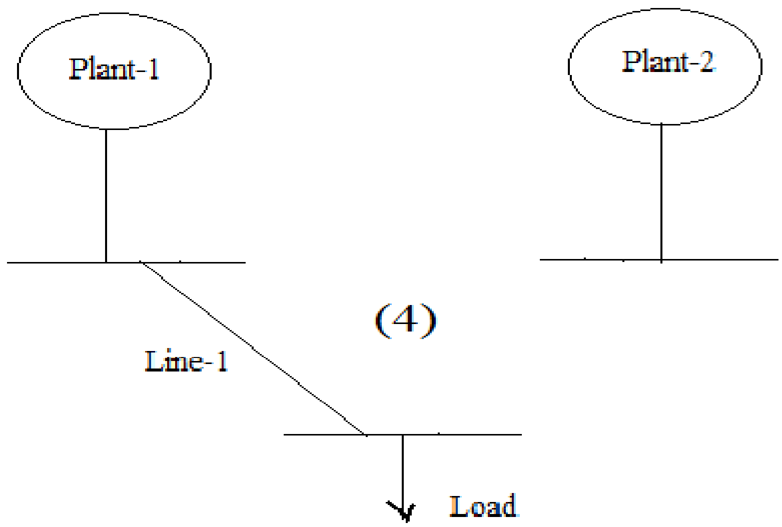
$P_c(3)$ is the curtailment probability for the system shown below.



For L1 In and L2 Out and L3 Out

$$Q_s = P_g(1) + P_c(4) - P_g(1).P_c(4)$$

$P_c(4)$ is obtained from



$P_g(1)$ represents the curtailment probability due to plant 1 supplying the entire load.

For L1 In

$$Q_s = R_{L2}[\{P_g(1,2) + P_c(1) - P_g(1,2).P_c(1)\}.R_{L3} + \{P_g(1,2) + P_c(2) - P_g(1,2).P_c(2)\}.Q_{L3}] \quad +$$
$$Q_{L2}[\{P_g(1,2) + P_c(3) - P_g(1,2).P_c(3)\}.R_{L3} + \{P_g(1) + P_c(4) - P_g(1).P_c(4)\}.Q_{L3}]$$

For L1 Out

$$Q_s = Q_s(L_2 - In).R_{L2} + Q_s(L_2 - Out).Q_{L2}$$

For L1 Out and and L2 In

$$Q_s = Q_s(L_3 - In).R_{L3} + Q_s(L_3 - Out).Q_{L3}$$

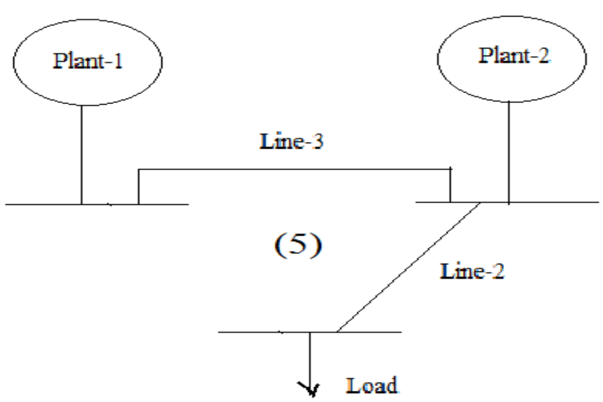
For L1 Out and L2 In and L3 In

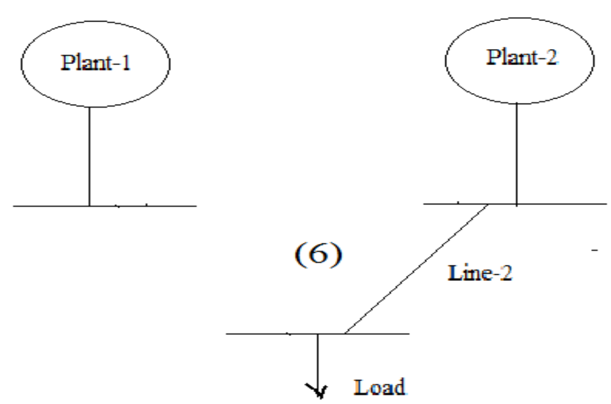
$$Q_s = P_g(1,2) + P_c(5) - P_g(1,2).P_c(5)$$

For L1 Out and L2 In and L3 Out

$$Q_s = P_g(2) + P_c(6) - P_g(2).P_c(6)$$

Configurations 5 and 6 are shown below.





For L1 Out and L2 In

$$Q_s = R_{L2}[\{P_g(1,2) + P_c(5) - P_g(1,2).P_c(5)\}.R_{L3} + \{P_g(2) + P_c(6) - P_g(2).P_c(6)\}.Q_{L3}] + Q_{L2}$$

$$Q_s = \{P_g(1,2) + P_c(1) - P_g(1,2).P_c(1)\}.R_{L3} + \{P_g(1,2) + P_c(2) - P_g(1,2).P_c(2)\}.Q_{L3}$$

$$R_{L2}[\{P_g(1,2) + P_c(5) - P_g(1,2).P_c(5)\}.R_{L3} + \{P_g(2) + P_c(6) - P_g(2).P_c(6)\}.Q_{L3}] + Q_{L2}$$

$$[R_{L3}(P_g(1,2) + P_c(1) - P_g(1,2).P_c(1)) + Q_{L3}(P_g(1,2) + P_c(2) - P_g(1,2).P_c(2))]$$
$$[R_{L3}(P_g(1,2) + P_c(3) - P_g(1,2).P_c(3) + Q_{L3}(P_g(1) + P_c(4) - P_g(1).P_c(4))]$$
$$[R_{L3}(P_g(1,2) + P_c(5) - P_g(1,2).P_c(5)) + Q_{L3}(P_g(2) + P_c(6) - P_g(2).P_c(6))]$$

$$Q_s = [\{P_g(1,2) + P_c(5) - P_g(1,2).P_c(5)\}.R_{L3} + \{P_g(2) + P_c(6) - P_g(2).P_c(6)\}.Q_{L3}]$$

For L1 Out and and L2Out

$$Q_s = 1.0$$

For L1 Out

$$Q_s = R_{L2}[\{P_g(1,2) + P_c(5) - P_g(1,2).P_c(5)\}.R_{L3} + \{P_g(2) + P_c(6) - P_g(2).P_c(6)\}.Q_{L3}] + Q_{L2}$$

$$Q_s = R_{L1}\{R_{L2}[R_{L3}\{P_g(1,2) + P_c(1) - P_g(1,2).P_c(1)\} + Q_{L3}\{P_g(1,2) + P_c(2) - P_g(1,2).P_c(2)\}]\} +$$
$$\{Q_{L2}[R_{L3}\{P_g(1,2) + P_c(3) - P_g(1,2).P_c(3)\} + Q_{L3}\{P_g(1) + P_c(4) - P_g(1).P_c(4)\}]\} +$$
$$Q_{L1}\{R_{L2}[R_{L3}\{P_g(1,2) + P_c(5) - P_g(1,2).P_c(5)\} + Q_{L3}\{P_g(2) + P_c(6) - P_g(2).P_c(6)\} + Q_{L2}]\}$$

Combining all the above, the complete expression for the system becomes

$$Q_s = R_{L1}\{R_{L2}[R_{L3}\{P_g(1,2) + P_c(1) - P_g(1,2).P_c(1)\} + Q_{L3}\{P_g(1,2) + P_c(2) - P_g(1,2).P_c(2)\}]\} +$$
$$\{Q_{L2}[R_{L3}\{P_g(1,2) + P_c(3) - P_g(1,2).P_c(3)\} + Q_{L3}\{P_g(1) + P_c(4) - P_g(1).P_c(4)\}]\} +$$
$$Q_{L1}\{R_{L2}[R_{L3}\{P_g(1,2) + P_c(5) - P_g(1,2).P_c(5)\} + Q_{L3}\{P_g(2) + P_c(6) - P_g(2).P_c(6)\} + Q_{L2}]\}$$

6.(b) Two power systems are interconnected by a 20 MW tie-line. System A has three 20 MW generating units with forced outage rates of 10%. System B has two 30 MW units with forced outage rates of 20%. Calculate the LOLE in system A for one-day period, given that the peak load in both system A and system B is 30 MW. **[12 Marks]**

Ref : 1) "Power System Reliability Evaluation"-Roy Billinton, Gordon & Breach Science Publishers-Pg-230

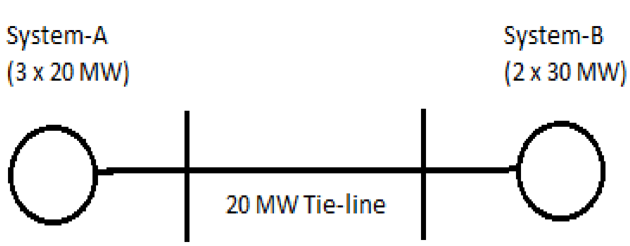


Table-1 (System-A)

Capacity In	Probability
0	$3C_0 * 0.90^0 * 0.10^3 = 1.0000E-3$
20	$3C_1 * 0.90^1 * 0.10^2 = 0.0270$
40	$3C_2 * 0.90^2 * 0.10^1 = 0.2430$
60	$3C_3 * 0.90^3 * 0.10^0 = 0.7290$

Table-2 (System-B)

Capacity In	Probability
0	$2C_0 * 0.80^0 * 0.20^2 = 0.04$
30	$2C_1 * 0.80^1 * 0.20^1 = 0.32$
60	$2C_2 * 0.80^2 * 0.20^0 = 0.64$

Table-1 is rearranged to form

Table-3 (System-A)

Capacity out	Probability
0	0.7290
20	0.2430
40	0.0270
60	0.0010

Likewise, Table-2 is rearranged to form

Table-4 (System-B)

Capacity out	Probability
0	0.64
30	0.32
60	0.04

Table-5

Probability of simultaneous outages in System-A & System-B

Capacity out		System-B 0	System-B 30	System-B 60
System-A	0	0.4466	0.2333	0.0292
System-A	20	0.1555	0.0778	9.72 E -3
System-A	40	0.0173	8.64 E -3	1.08 E -3
System-A	60	6.4 E -4	3.2 E -4	4 E -5

The entries in the above table are obtained from Tables-3 & 4.
Eg : [40,30] = 0.027 * 0.32 = 8.64 E -3

Table-6

Loss of load in System-A / Load loss array for System-A (since the question is for System-A only)

System-A details
Installed capacity = 3 x 20 MW
Peak load = 30 MW (Given in Question)
Reserve capacity = 20 MW (**assumed to be the highest rating**)
Tie capacity = 20 MW (given)

System-B details
Installed capacity = 2 x 30 MW
Peak load = 30 MW (Given in Question)
Reserve capacity = 30 MW (**assumed to be the highest rating**)
Tie capacity = 20 MW (given)

Capacity out		System-B 0	System-B 30	System-B 60
System-A	0	0	0	0
System-A	20	0	0	0
System-A	40	10	20	20
System-A	60	30	40	40

Entry details

[0, 0] = 0(Outage) - 20(Reserve) - 10(From B) = 0

[20, 0] = 20(Outage) - 20(Reserve) - 0(From B) = 0

[20,30] = 20(Outage) - 20(Reserve) - 0(From B) = 0

[20,60] = 20(Outage) - 20(Reserve) - 10(From B) = 0

[40, 0] = 40(Outage) - 20(Reserve) - 10(From B) = 10

[40,30] = 40(Outage) - 20(Reserve) - 0(From B) = 20

[40,60] = 40(Outage) - 20(Reserve) - 0(From B) = 20

[60, 0] = 60(Outage) - 20(Reserve) - 10(From B) = 30

[60,30] = 60(Outage) - 20(Reserve) - 0(From B) = 40

[60,60] = 60(Outage) - 20(Reserve) - 0(From B) = 40

Reproducing Table-6 below. We select only those entries which have non-zero load loss. Thus Table-7 is created.

Capacity out		**System-B** 0	30	60
System-A	0	0	0	0
	20	0	0	0
	40	10	20	20
	60	30	40	40

Table-7 (System-A)

Capacity out		**System-B** 0	30	60
System-A	0			
	20			
	40	0.0173	8.64 E -3	1.08 E - 3
	60	6.4 E -4	3.2 E -4	4 E -5

Therefore, expected load loss probability =
Σ Prob. = 0.0173 + 8.64 E -3 + 1.08 E -3 + 6.4 E -4 + 3.2 E -4 + 4 E -5
Σ Prob. = 0.02802 LOLE (Loss of load expectation)
7. Write short notes on the following.

(a)	Loss of load approach	[7 Marks]
(b)	Frequency and duration approach	[7 Marks]
(c)	Multiple bridge equivalents	[6 Marks]

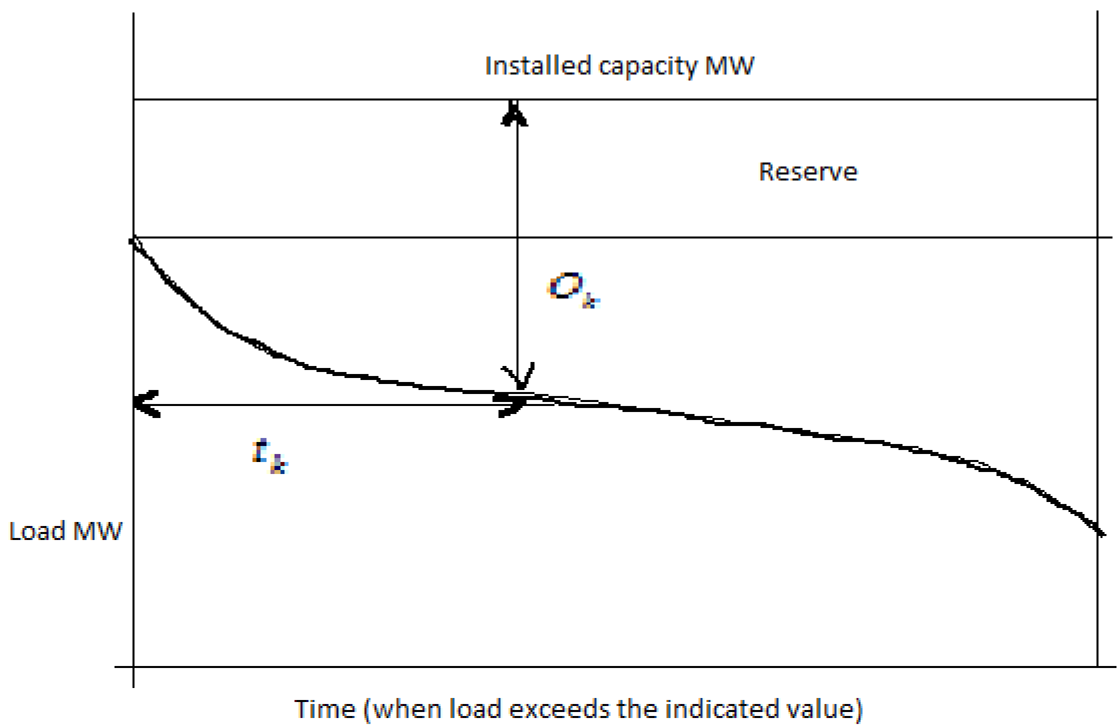
[5 x 20 = 100 Marks]

7A Loss of load approach

Ref : 1) "Power System Reliability Evaluation"-Roy Billinton, Gordon & Breach Science Publishers-Pg-104,
2) "Reliability Modeling in Electric Power Systems"- J.Endrenyi, Wiley,Pg-120,
3) "Power System Reliability, Safety and Management", Balbir Singh Dhillon, Ann Arbor Science, Page-202

In this approach, the applicable system capacity outage probability table is combined with the system characteristics to give an expected risk of loss-of-load. The units are in days or hours depending upon the load characteristics used. Prior to combining the outage probability table, the difference between the terms "capacity outage" and "loss of load". The term "capacity outage" indicates a loss of generation which may or may not result in a loss of load. This condition depends upon the generating capacity reserve margin and the system load level. However, "loss of load" occurs only when the capability of the generating capacity remaining in service is exceeded by the system load level.
A typical system load capacity relationship is shown in Fig.23

Fig. 23 Relationship between Load, Capacity and Reserve



Ok = Magnitude of the k th outage in the system capacity probability table

Pk = Probability of an outage of capacity equal to Ok.

t_k = No. of times , units in the selected interval that an outage magnitude Ok would cause a loss of load.

A particular capacity outage will contribute to the system expected load loss by an amount equal to the product of the probability of existence of the particular outage and the number of time units in the selected interval that loss of load would occur. (If such a capacity outage were to occur.)

From Fig. 23, it is evident that any capacity outage less than the reserve will not contribute to the system expected load loss. Outages of capacity in excess of the reserve will result in varying numbers of time units during which load loss could occur. This is expressed mathematically as $E(t) = \sum_{k=1}^{n} P_k t_k Timeunits$

$$E(t) = \sum_{k=1}^{n} P_k t_k Timeunits$$

If the load characteristics in the above figure is the load duration curve, the loss of load expectance is in hours. If a daily peak load variation curve is used, the loss of load expectancy is in days for the period of study.

7B Frequency and duration approach

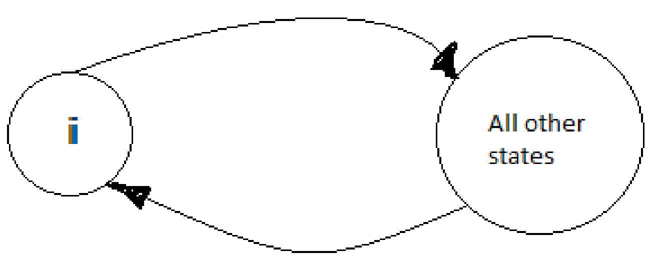
Ref : 1) "Reliability Modeling in Electric Power Systems"- J.Endrenyi, Wiley, Pg-53

State frequencies and durations

The frequency of encountering state i, f_i, is defined as the expected number of stays in (or arrivals into, or departures from) i per unit time, computed over a long period. By this definition, the concept of frequency is associated with the long term behavior of the process describing the system. The mean duration of the stays in state i must also be computed over a long period of time.

In order to relate the frequency, probability, and mean duration of a given system state, the history of the system will be regarded as consisting of two alternating periods, the stays in i and the stays outside i. Thus the system is represented by a two-state process whose state-space diagram is shown in Fig.dddd. Let the mean duration of the stays in state i be T_i and that of the stays outside I, be $T_i^{'}$.

Fig.- 7.1 Two-state process



The mean cycle time , $T_{ci} = T_i + T_i^{'}$

From the definition of the state frequency, it follows that in the long run, f_i equals the reciprocal of the mean cycle time.

$$f_i = \frac{1}{T_{ci}}$$

Multiplying the above eqn. by T_i, we have $p_i = \frac{T_i}{T_{ci}}$

When we analyze the long-term behavior of a component, its probability of being in the up-state is the ratio of the mean up-time to the sum of the mean up and down-times (proportion of the time that the component is working). Similarly its probability of being in the down-state is the ratio of the mean down-time to the sum of the mean up and down-times (proportion of the time that the component is not working).

$$p_i = \frac{T_i}{T_{ci}}$$

Hence $f_i T_i = p_i$ where p_i represents the probability of being in state i.

This is a fundamental equation which provides the relation between the three parameters.

Next, the frequencies f_i, mean durations T_i and the transition rates in the system will be related. To begin with, the concept of the frequency of transfer from state i to state j is introduced. This frequency f_{ij} is introduced. This frequency f_{ij} is defined as the expected number of direct transfers from i to j per unit time.
We know that the intensity of transition from state i to state j, $q_{ij}(t)$ is defined as

$$q_{ij}(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P[X(t+\Delta t) = j \mid X(t) = i])$$

Where X(t) is the random variable representing the system state at time t, and similarly for $X(t+\Delta t)$

$$f_{ij} = \lim_{\Delta t \to 0} \frac{1}{\Delta t} P[(X(t+\Delta t) = j) \cap (x(t) = i)]$$

$$= \lim_{\Delta t \to 0} \frac{1}{\Delta t} P[X(t+\Delta t) = j \mid x(t) = i] P[X(t) = i]$$

$$= \lambda_{ij} p_i$$ where λ_{ij} is the transition rate

Thus, λ_{ij} is essentially a conditional frequency, the condition being that the system resides in *i*. Now, from the definitions of f_i and f_{ij} it follows, that:

$$f_i = \sum_{j \neq i} f_{ij}.$$

Hence, $f_i = p_i \sum_{j \neq i} \lambda_{ij}$.

We already know that $f_i T_i = p_i$

Hence, $T_i = \dfrac{1}{\sum_{j \neq i} \lambda_{ij}}$

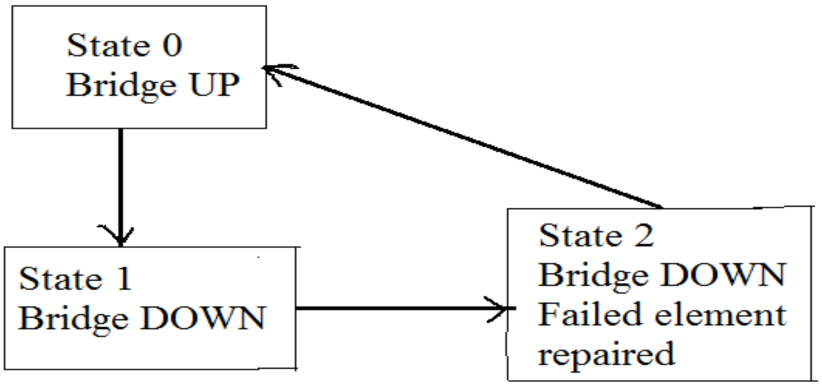
Thus it can be inferred that the mean duration of the stays in any given state equals the reciprocal of the total rate of departures from that state. All the state indices can now be computed from the transition rates that define a given system.

(c) Multiple bridge equivalents [6 Marks]

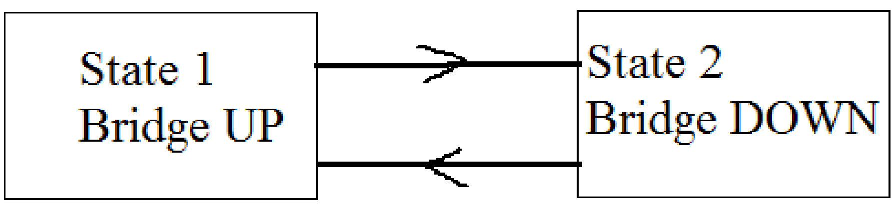
Ref : 1) "Power System Reliability Evaluation"-Roy Billinton, Gordon & Breach Science Publishers-Pg-266

The concept of bridge equivalents occurs in the context of DC transmission lines. The bridge is a fully controlled one. It has repairable elements (SCR devices).

The state space diagram of a single bridge is shown below.

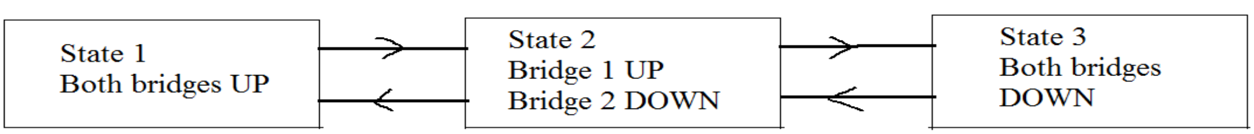


The above can be approximated by an equivalent model.



In a multiple bridge, two or more bridges are involved. Each has its own set of repairable elements. In the case of two bridges, both are required for system success.
Then the equivalent state space representation is a binary state model similar to the one obtained for a single bridge.

<u>Equivalent model for two bridges</u>



In addition, the state space diagram for two bridges are represented with derating.