F 9288 (Pages : 3) Reg. No.................................

Name.....................................

B.TECH. DEGREE EXAMINATION, NOVEMBER 2011

Third Semester

Branch—Computer Science/Information Technology

ENO 10 301 B—ENGINEERING MATHEMATICS—II (CS, IT)

(Regular)

Time : Three Hours Maximum : 100 Marks

Part A

Answer **all** *questions briefly.*
Each question carries 3 marks.

1. Write in symbolic form :
   - (a) Some girls are not white.
   - (b) It is true that all roads lead to Kollam
   - (c) Some cones are not good.
2. Using Euclidean algorithm, find gcd of 15276 and 2055.
3. Give examples of two functions $f: N \to Z$ and $g: Z \to Z$ such that $g \circ f$ is injective but g is not injective.
4. Define a Bounded lattice and a Sublattice.
5. Define
   - (a) Hamiltonian cycle.
   - (b) Spanning tree.

(5 × 3 = 15 marks)

Part B

Answer **all** *questions, each question carries 5 marks.*

6. Construct truth table for $P \vee \neg(P \wedge Q)$.
7. If $a \equiv b (\text{mod } n)$ then show that $a^k = b^k (\text{mod } n)$ for every positive integer k.
8. I denotes the set of all integers and m is an integer. Show $R = \{ < x, y > / x - y \text{ is divisible by } m\}$ is an equivalence relation.
9. Define chain and subchains and show that every chain is a distribution lattice.
10. Give an example of a graph in which the length of the longest cycle is 9 and the length of the shortest cycle is 4.

(5 × 5 = 25 marks)

Turn over

Part C

*Answer any **one** full question from each module.*
Each full question carries 12 marks.

Module 1

11. Show that :

(a) $(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$. (6 marks)

(b) $(\exists x)(M(y) \wedge \neg W(y))$ if (x) $(F(x) \rightarrow \neg S(x))$ follows. (6 marks)

Or

12. (a) Show that $(\forall x)(P(x) \wedge Q(x)) \rightleftarrows ((\forall x)P(x)) \wedge ((\forall x)Q(x))$ is a logically valid statement. (6 marks)

(b) Show the following implications without constructing truth tables.

$(P \rightarrow Q) \vee (R \Leftrightarrow P) \wedge (Q \vee R)$. (6 marks)

Module 2

13. (a) If a/c and b/c then prove that $gcd\ (a, b)/c$. (5 marks)

(b) If p is a prime, then prove that $a^p \equiv a (\text{mod } p)$. (7 marks)

Or

14. (a) Show that the functions f and g which both are from N × N to N given by $f(x, y) = x + y$ and $g(x, y) = x\ y$ are onto but not one-to-one. (6 marks)

(b) Check whether the following functions are invertible. If so, compute the inverse :

(i) $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = |x|, \forall x \in \mathbb{R}$. (3 marks)

(ii) $g : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $g(x) = 2x - 1, \ \forall x \in \mathbb{R}$. (3 marks)

Module 3

15. (a) Show that the "set inclusiong $\subseteq$" is a partial ordering on power set of A for any set A. (6 marks)

(b) If relations R and S are reflexive, symmetric and transitive, show that $R \cap S$ is also reflextive, symmetric and transitive. (6 marks)

Or

16. (a) Define an equivalence relation. If ~ is an equivalence relation of a set X, show that the corresponding equivalence classes form a portion of X. (6 marks)

(b) Define partial order and total order relations. Give an example of a partial order which is not a total order and also *vice versa*. (6 marks)

Module 4

17. (a) If < L, * ⊕ > is a distributive lattice, then prove for any $a, b, c \in$ L,

$$(a * b = a * c) \wedge (a \oplus b = a \oplus c) \Rightarrow b = c.$$ (6 marks)

(b) Define a complete lattice and complemented lattice. Draw the Hasse diagram for D_{40}, the lattice of all positive divisors of 40. (6 marks)

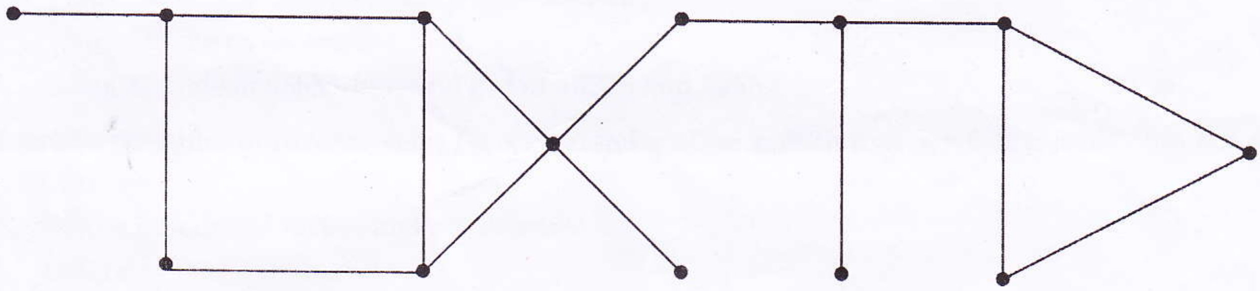
Or

18. (a) In a lattice $< L, \leq >$ with $a, b, c \in L$, show that $a \leq c \Rightarrow a \oplus (b * c) \leq (a \oplus b) * c$. (6 marks)

(b) Which of the two lattices < S_n, D > for n = 30 and n = 45 are complemented ? Prove whether they are distributive. (6 marks)

Module 5

19. Let G be the graph shown below :



(a) Find a closed walk of length 6. Is your walk a trial ? (2 marks)

(b) Find an open walk of length 12. Is your walk a path ? (2 marks)

(c) Find a closed trial of length 6. Is your trial a cycle ? (2 marks)

(d) What is the length of the longest cycle in G ? (2 marks)

(e) What is the length of a longest path in G ? How many paths are there of this length ? (4 marks)

Or

20. (a) Draw all non-label-isomorphic graphs with three vertices using the label set V = {a, b, c}. (6 marks)

(b) If G be a connected graph which is not a tree and let C be a cycle in G. Prove that the complement of any spanning tree of G contains at least one edge of C. (6 marks)

[5 × 12 = 60 marks]